# THE INVESTIGATION OF TWO SUBSONIC TYPE AIRFOILS IN THE TRANSONIC REGION BY APPLICATION OF THE HYDRAULIC ANALOGY

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# LIST OF SYMBOLS AND ABBREVIATIONS

| Symbol | Definition |
|---|---|
| $a$ | (a) Speed of sound; (b) Airfoil camber factor |
| $c$ | (a) Celerity, velocity of disturbance propogation; (b) chord |
| $c_d$ | Section drag coefficient |
| $c_l$ | Section lift coefficient |
| $c_{m_{1/4}}$ | Section moment coefficient about the quarter chord point |
| $C_c$ | Chord force coefficient, parallel to chord line |
| $C_N$ | Normal force coefficient (normal to chord line) |
| $C_p$ | Pressure coefficient, $\frac{\Delta p}{q} = -\frac{p_s - p_l}{q}$ |
| $c_p$ | Specific heat at constant pressure |
| $c_v$ | Specific heat at constant volume |
| $d$ | Depth of fluid |
| $g$ | Acceleration of gravity (assumed to be 32.2 ft/sec$^2$) |
| $K$ | (a) Drag due to lift factor, $c_{d_l}/c_l^2$ ; (b) Similarity constant |
| $M$ | Mach number, velocity/speed of sound |
| $p$ | Pressure |
| $q$ | Dynamic pressure, $\frac{1}{2}\rho_s V_s^2$ or $\frac{\gamma}{2} p_s M_s^2$ |
| $t$ | Thickness (perpendicular to flow) |
| $T$ | (a) Temperature; (b) Surface tension |
| $u$ | Velocity, parallel to free stream flow |
| $v$ | Velocity perpendicular to free stream flow (two dimensional) |
| $V$ | Absolute velocity, $\sqrt{u^2 + v^2}$ |

| | |
|---|---|
| $\alpha$ | Angle of attack |
| $\gamma$ | Ratio of specific heats of a gas, $C_p / C_v$ |
| $\lambda$ | Wave length |
| $\varphi$ | Velocity potential |
| $\rho$ | Mass specific density |

Subscripts

| | |
|---|---|
| $cr$ | Critical, value of factor where M = 1.0 |
| $\ell$ | Local conditions |
| $LO$ | Value at zero lift |
| $min$ | Minimum value |
| $o$ | Stagnation or resevoir conditions |
| $PE$ | Parabolic estimation |
| $s$ | Free stream conditions |
| $x, xx$ | First and second derivatives in the u direction |
| $y, yy$ | First and second derivatives in the v direction |
| $ZL$ | Value at zero lift |
| $\alpha$ | First derivative with respect to angle of attack |

## SUMMARY

A series of experiments were conducted at the Georgia Institute of Technology to determine the applicability of the hydraulic analogy in the transonic testing of subsonic type airfoils. Two roundnose airfoils of sixteen per cent maximum thickness, but of different shape, were tested in order to compare the effect of shape on the behavior of pressure distribution and lift, drag and moment coefficients in the transonic region.

The airfoils, the Goldstein 1442/1547 and the Gottingen 624, were tested at six angles of attack from - 2.5° to + 6.5° and at six Mach numbers from 0.57 to 1.47. Results were compared with wind-tunnel data at the extremes of the velocity region to ascertain the degree of correlation possible with the analogy. Qualitative comparisons of the airfoils were possible over a major portion of the velocity region.

The hydraulic analogy, wherein free surface liquid flow is analogous to the two-dimensional flow of a compressible gas, is easily applied to experimental work. The water channel was found to be a desirable test device. The water channel and associated equipment comprise a minimum of equipment when compared to the wind-tunnel. The analogy is simple to apply experimentally and the channel is easily operated.

It was found that the degree of correlation obtained using the analogy with wind-tunnel data varied with test Mach number. The

comparison of water channel and wind-tunnel results, applying the analogy directly and with a modification, formulated the following conclusions.

1. At supersonic Mach numbers the degree of correlation is high. Water channel data is in close agreement with wind-tunnel results.

2. Water channel data is, at best, qualitative in nature at subsonic speeds when using airfoils of the type tested in this thesis.

3. Further investigation is necessary in order to ascertain effects of viscosity, surface tension and boundary interference in water channel data.

4. Research is required to fully determine the effects of difference in specific heat ratios in transonic gas flow.

A comparison of the behavior of the two airfoils indicates that airfoil shape is of considerable importance in the transonic region. A qualitative comparison demonstrates two important facts.

1. The airfoil designed for high subsonic speeds (the Goldstein 1442/1547) also has more desirable characteristics at high transonic and supersonic velocities than the low subsonic type (Gottingen 624).

2. The effects of camber decrease at supersonic Mach numbers.

# CHAPTER I

## INTRODUCTION

Before the advent of World War II only theorists and a few experimentalists were interested in transonic and supersonic flight. However, the war caused such rapid development in aircraft and their engines that before the end of the conflict designers were faced with entirely new problems in aerodynamics. Although supersonic flight, once achieved, has many problems for the designer, the major difficulties lie in the velocity regime where compressibility effects are first noticed up to the point where all flow is supersonic. This region is hard to define and differs for each body in flight. It is in this region that very little is known at the present time.

The very nature of the air flow in the transonic region makes theoretical work very difficult and experimental work doubtful. This "mixed flow", partially subsonic, part supersonic, cannot usually be predicted by application of present accepted theories. These theories deal mostly with three types of flow; incompressible subsonic, compressible subsonic and compressible supersonic. The few theories dealing with mixed flow are so limited in scope at the present time, that application to most practical problems is extremely laborious, if not actually impossible.

Because of the difficulties in solving transonic flow fields by theoretical means, designers have been forced to rely on experiment.

Ever since the first "buffeting", "tuck under" and "drag rise" phenomena were encountered near the "sonic barrier", testing methods have been devised, or attempted, to give corresponding results on test aircraft or models. These methods, to name a few are: fired missiles with models attached, models mounted to high speed aircraft, free fall models and modification of existing subsonic tunnels. The problems of accurate measuring devices and limitation of the types of study made are evident in the first three testing devices mentioned. Accurate wind-tunnel testing at transonic speeds is to be desired. The number of factors that can be observed and measured in this method are large compared to the other procedures.

Since the wind-tunnel has been established as a standard testing device, for both subsonic and supersonic flow, it would be well to examine the possibilities of the wind-tunnel for transonic flow. However, the problem arises in obtaining free flight conditions in a wind-tunnel. There are four major problems to be met in wind-tunnel testing at transonic speeds. These are briefly stated below.

(a) Establishment of transonic flow is a primary consideration. While the power required is naturally large one of the main problems is in the extreme care that must be taken in area consideration. Boundary layer thickness, physical dimensions of the tunnel, anything that can affect the area ratios must be carefully controlled. The physical characteristics of the air itself, down stream and/or upstream conditions are important.

(b) The blocking or chocking effect of the model can be considered separately, but has a lot to do with the initial establishment of the flow.

The model must be kept extremely small to keep from forming a shock in front of the model that completely chokes the tunnel or limits the testing Mach number.

(c) Boundary effect is another major problem. The pressures and therefore the forces on the model are affected by any alteration of the flow caused by the tunnel walls. Corrections for this effect are still being developed for subsonic and supersonic testing and transonic flow offers new problems.

(d) Shock wave reflection from the tunnel walls also causes concern. The reflection from a shock preceeding the model at supersonic speeds or from shocks from some point on the model itself may completely disrupt the flow on part or all of the model, causing any observations to be in error.

The answer to at least the latter three is the extremely small model. But here the problem of accurate measurement of extremely small forces and the effect of supports becomes predominate. Problems such as noise, danger to operator, building, operating and maintaining the tunnel are considerable but can usually be solved without great difficulty and lavish use of money.

Three methods of wind-tunnel testing are in use for transonic work. The flexible wall tunnel can be of great assistance in establishing high subsonic Mach numbers and decreasing interference effects. The bump method of accelerating the flow is used a great deal but requires three dimensional reflection type model of small size and does not give a constant Mach number over the span of the model. It does, however, use standard high speed tunnels. Recent developments are the slotted throat and porous wall tunnels that alleviate many of the aforementioned problems. These devices are still in their infancy and the results from these tunnels are still doubtful.

That the transonic region is important is obvious. Until greater propulsive power is available, there is a physical limit to practical speeds. Only a few experimental aircraft are above transonic velocities at the present time. Future supersonic aircraft must pass through the sonic speed in at least two phases of their flight and long range flight will probably be high subsonic in nature. It is evident that testing methods need to be established for this velocity range.

The use of analogies can often be of great value in the solution of a problem. There are two analogies to compressible flow in gases. They are the electrical and the hydraulic. It is the latter that is the concern of this thesis. The theory and application of the hydraulic analogy is explained in another section. This analogy has been shown in previous works to be of great value in simulated supersonic testing. The simplicity of construction and ease of operation of testing devices and the low test speeds involved all add to its practicability. Even though its use is limited to the two-dimensional case results could be of great value, even if of a qualitative nature.

The purpose of this thesis can be said to be twofold. First, to investigate the use of the hydraulic analogy as a test device for the transonic region. Second, to use the analogy to compare the behavior of two thick airfoils of different shape but equal maximum thickness. The thick airfoil is desirable structurally and may offer a solution to other problems of near-sonic flight. Such theory that exists concerns thin or perfect geometric bodies only. Wind-tunnel results are meager and doubtful. In view of this, it is hoped that the results of the research contained in this thesis will establish an easier method of transonic testing and supply data that can be supplemented to form an extensive study in a field where little work has yet been done.

# CHAPTER II

## TRANSONIC FLOW

Theoretical.--Precisely speaking, air flow is compressible, viscous, neither adiabatic, isentropic or irrotational in nature. To derive flow equations of a practical nature involving all variables and satisfying all conditions has been, to date, an insurmountable task. However, certain types of flow may be analized theoretically by making simplifying assumptions that do not affect results to a great extent. The types or classes of gas flow that are treated in this manner and yield applicable results within specified bounds are: (a) incompressible, non-viscous, (b) incompressible viscous and (c) compressible, non-viscous. Several methods of analyzing flow in each of the types have been developed, the complexity of the method depending upon the limitations.

Until about fifteen years ago subsonic flow was of utmost importance. The non-linear equations of flow were simplified by assuming air to be a perfect gas, incompressible and irrotational in nature. Resulting linear differential equations could be used with arbitrary airfoils to predict flow patterns, pressure coefficients and other aerodynamic coefficients with very good results. The study of viscous flows and resulting boundary layer build-up has undergone extensive study. In this type of flow, Reynolds number is the important similarity parameter.

The consideration of compressible flow presents flow equations that must be linearized differently than the equations of incompressible flow. Subsonic flow was linearized in part by neglecting compressibility. Ackeret was one of the first to linearize the supersonic equations. His method was to apply the small pertubation method to flow problems. This method implies that velocity changes around a body are small in comparison to the velocities far from the body and sonic velocities. This theory therefore has the proviso of slender bodies.

The Chaplygin hodograph method does not linearize the flow equations but does present a usable method. Usable in a restricted sense, however. Arbitrary shapes can only, with the greatest of difficulty, be transformed to the hodograph plane. Thus flow problems may be solved with a good degree of accuracy for supersonic and low subsonic flow. However, compressibility effects cannot be ignored for high subsonic velocities. The basic assumption of incompressibility must fail when the flow begins to be affected by compressibility, density variations and shock waves. As the Mach number approaches unity (flow of sonic speeds) the small perturbation method fails also, even for very thin bodies. The Prandtl-Glauert correction factor $\sqrt{1-M^2}$ is hardly applicable at near sonic velocities. Neither can the Karman-Tsein hodograph method be applied since $\sqrt{1-M^2}$ is also involved in this solution, Ref. 1.

The foregoing paragraph has partially outlined the transonic flow problem. Before proceeding the types of transonic flow will be described. The word "transonic" defines this type of flow very aptly as those velocities that span sonic speed. This transonic region may be broken into three types where flow over a body is concerned, Fig. 1.

The first compressibility effects usually occur when the velocity over the airfoil reaches sonic speed at some point. The free stream velocity at which this first occurs may well be called the "lower critical Mach number" as suggested by von Karman. Fig. 1 (a) illustrates this flow.

At some free stream subsonic velocity flow around a body cannot exist without the presence of a discontinuity (shock wave), Fig. 1 (b). This region is known as the "limiting Mach number" and may exist even in a non-viscous fluid.

At supersonic free stream velocities bodies having other than sharply pointed leading edges will have a curved detached shock preceeding them, Fig. 1 (c). There is a subsonic region behind the normal and near normal portion of the curved shock surrounding the leading edge of the body. The extent of this region depends on the shape of the body and the free stream Mach number.

As could well be expected with a mixed flow regime, the theory concerning it is in somewhat confused state. Prandtl and Glauert, Karman and Tsein were among the first to propound corrections for compressibility effects. These dealt primarily with conditions involving the lower critical Mach number. Based on some form of linearization of the flow equations, their corrections break down at transonic velocities due to the mixed character of the flow. Some degree of non-linearity must be kept in order to obtain non-trivial results. The present approach is that of dimensional analysis, the results known as the various similarity laws or rules. Here the flow equation is kept with all the pertinent variables. The equation may be considered to have "degrees of freedom" equal to the number of variables less the number of basic flow equations that must be satisfied. Simplifying assumptions must still be made, but their magnitude is not as great as before. In all cases to date two restrictions have been put on the usage of the similarity laws:

1. Only those velocities very near sonic speeds may be considered. This region is approximately M = 0.8 to 1.2 or less.

2. Only small thickness/length ratios were used. Airfoils of six per cent or less thickness ratio are usually considered.

To obtain mechanically similar flow subsonically Reynolds number is the similarity parameter. To obtain similar flow supersonically Mach number is used. The results of transonic flow dimensionless analysis yield a transonic similarity parameter. As many of these parameter exist as authors who have examined the problems. Karman, Kaplan, Busemann (Refs. 2, 3, and 4 respectively), and several others have examined the transonic similarity conditions and arrived at somewhat different results. However, all are similar in the applications and variables treated.

Karman's similarity parameter, (Ref. 1), is

$$K = \left[\frac{M^2-1}{\left(\frac{t}{c}\right)^{\frac{2}{3}}}\right]$$

Kaplan, Ref. 2, shows it to be

$$K = \frac{1-M_\infty}{\left[(\gamma+1)\frac{t}{c}\right]^{\frac{2}{3}}}$$

Busemann, Ref. 3, uses a disturbance velocity potential, $\varphi$ , to develop a "parabolic formula" using the basic equation

$$\varphi_{xx} + \varphi_{yy} = 0$$

He states a warning to the scientist in these words:

> As long as similarity is used as a sideline for theory, its limitations may not be exceeded except for explanatory purposes. As a tool of experimental research, similarity may more likely vary between too narrow and too broad applications, or it may become fixed to a standard form until its generality is rediscovered from time to time.

Actual computation of pressure coefficients by theoretical methods in transonic flow is definitely in its infancy. The detached shock, and its associated problems has been considered by Ferri in Ref. 22 and others. The interaction of boundary layer and shock waves presents other problems which have been essentially solved.

Experimental.--Experimental research in the transonic region is, in the main, of a classified nature at the present time. The problems associated with experimental transonic research have been stated previously in the Introduction. However, there is available enough information to draw several conclusions:

1. As demonstrated by Liepman, Ref. 4 the type of flow, laminar or turbulent has considerable effect on pressure distribution and shock wave formation.

2. The results in Ref. 5 shows that for the airfoils tested the section drag varies little with maximum thickness location. Lift-curve slope, the angle of zero lift are adversely affected, however. Also shown in this report is the fact that drag may be reduced only by

decreasing the $\frac{t}{c}$ ratio. The statement concerning drag is open to question.

3. Enough data is available to establish similarity parameters for transonic flow. This applies to both two and three dimensional cases, but can be applied only in restricted cases.

4. The methods of obtaining transonic data leave much to be desired and results must be carefully examined before application.

# CHAPTER III

## THE HYDRAULIC ANALOGY

Historical.--The use of analogies has always been of importance in scientific endeavor. Simplification of both experimental and theoretical problems may be accomplished through similitudes. In the hydraulic analogy to compressible gas flow the similarity of the flow of a free surface liquid to the two-dimensional compressible flow of a gas offers a possible solution to several of the problems encountered in experimental aerodynamics. The application of this analogy involves the flow of a shallow liquid past an immersed model. The depth and velocity of the fluid determine the flow parameter corresponding to Mach number in gas flow. The advantage of the analogy is that, to obtain agreement of flow parameters, the velocity of the liquid is much less than that of the gas. Thus the many problems associated with high velocity gas flow can be sidestepped in favor of a low speed liquid with the resulting savings in power, complexity, etc.

The idea of the hydraulic analogy first appeared in published form in 1920 when Jouquet, Ref. 6, mentioned the analogy and its possibilities. The mathematical basis was presented by Riabouchinsky, Ref. 7, in which he also described his apparatus for investigating the flow in a Laval nozzle. He extended the theory to include drag considerations and outlined the probable usefulness of the hydraulic analogy, Ref. 8. In Ref. 9 Binnie and Hooker made surveys along the center line of a channel with a constriction. Preiswerk's theoretical investigation, Ref. 10, together with his use of the method of characteristics to solve flow in a Laval nozzle proved conclusively that the methods of gas dynamics could be applied to water flow with a free surface.

In Ref. 11 Orlin, Lindner and Bitterly conducted photographic and depth studies for flow about circular cylinders. These studies together with work by Laitone, Ref. 12, and Harleman, Ref. 13, further substantiates the use of the analogy as considered in these reports.

Several assumptions must be made in the mathematical development of the analogy.

1. The flow is irrotational

2. The vertical accelerations in the fluid are negligible compared with the acceleration of gravity. The pressure in the fluid at any point depends only on the height of the free surface above that point.

3. The flow is steady. (Although the analogy can be applied to accelerated motion).

4. The fluid is incompressible.

5. Pressure at the free surface is constant.

6. Surface tension forces are negligible.

7. The flow is frictionless and adiabatic.

8. The gas is polytropic; internal energy is a function of temperature alone.

The first and last assumptions cause very little trouble in establishing the analogy. The fourth and fifth are accepted as basic assumptions in hydraulics. Vertical accelerations are usually low for the flow speeds involved. Surface tension and friction effects will be discussed later in the chapter Discussion.

The theory and mathematical development of the hydraulic analogy may be found in Refs. 10 and 13. The following is a condensation of these presentations.

<u>Mathematical derivation</u>.--The fundamental equations of flow for a free surface liquid and a gas may be developed in order to establish the analogy. They are compared in equations 1 through 6.

The continuity equations:

$$\frac{\partial(ud)}{\partial x} + \frac{\partial(vd)}{\partial y} = 0 \qquad \text{Free Surface Liquid} \qquad (1)$$

$$\frac{\partial(u\rho)}{\partial x} + \frac{\partial(v\rho)}{\partial y} = 0 \qquad \text{Gas} \qquad (2)$$

The energy equations:

$$V^2 = 2g(d_o - d) \qquad (3a)$$

$$V_{max} = \sqrt{2gd_o} \qquad \text{Liquid} \qquad (3b)$$

$$V^2 = 2g\,C_p(T_o - T) \qquad (4a)$$

$$V_{max} = \sqrt{2g\,C_p T_o} \qquad \text{Gas} \qquad (4b)$$

The velocity potential differential equation:

$$\Phi_{xx}\left(1 - \frac{\Phi_x^2}{gd}\right) + \Phi_{yy}\left(1 - \frac{\Phi_y^2}{gd}\right) - 2\Phi_{xy}\frac{\Phi_x \Phi_y}{gd} = 0 \qquad \text{Liquid} \qquad (5)$$

$$\Phi_{xx}\left(1 - \frac{\Phi_x^2}{a^2}\right) + \Phi_{yy}\left(1 - \frac{\Phi_y^2}{a^2}\right) - 2\Phi_{xy}\frac{\Phi_x \Phi_y}{a^2} \qquad \text{Gas} \qquad (6)$$

From equations (1) and (2) the analogy

$$\frac{d}{d_o} = \frac{\rho}{\rho_o} \tag{7}$$

can be derived.

If from equations (3) and (4) $V/V_{max}$ for the liquid is equated to $V/V_{max}$ for a gas

$$\frac{d_o - d}{d_o} = \frac{T_o - T}{T_o} \quad \text{or} \quad \frac{d}{d_o} = \frac{T}{T_o} \tag{8}$$

For adiabatic, isentropic flow of a gas

$$\frac{\rho}{\rho_o} = \left(\frac{T}{T_o}\right)^{\frac{1}{\gamma-1}} \tag{9}$$

and since from the analogy

$$\frac{\rho}{\rho_o} = \frac{d}{d_o} = \frac{T}{T_o}$$

then

$$\left(\frac{T}{T_o}\right)^{\frac{1}{\gamma-1}} = \frac{T}{T_o} \qquad \text{and} \quad \gamma = 2.0$$

From the relationship

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^{\gamma} \tag{10}$$

it is seen that

$$\frac{p}{p_o} = \left(\frac{d}{d_o}\right)^2 \tag{11}$$

The velocity potential equations (5) and (6) show that

$$g\,d = a^2 \tag{12}$$

Equation (12) shows the velocity $\sqrt{g\,d}$ corresponds to the velocity of sound in gas flow. In the Discussion section it is shown that $\sqrt{g\,d}$ may be considered the velocity of propagation of surface waves. This statement is true only if the wave length of the surface waves is large compared to the water depth.

The ratio $V/\sqrt{gd_s}$ known as Froude number in hydrodynamics corresponds to the Mach number, V/a, in aerodynamics. If the velocity is less than $\sqrt{gd_s}$, $(M<1)$ the liquid is said to be "streaming". If the flow is greater than $\sqrt{gd_s}$, $(M>1)$ the water is said to be "shooting".

It may be shown that under certain conditions, in shooting water (and only in shooting water) a discontinuity in the flow may occur. This discontinuity wherein the velocity of the flow decelerates to "streaming" and the depth increases is known as a hydraulic jump. Hydraulic jumps of small intensity are propogated with the velocity $\sqrt{gd_s}$ and correspond to sound waves. Harleman, Ref. 13 has a rigorous development of the analogy for normal and oblique jumps in fluid flow to the shock waves in gases.

The analogy may be summarized as follows:

| Compressible Gas Flow | Free Surface Liquid Flow |
|---|---|
| Temperature ratio, $\frac{T}{T_o}$ | Water depth ratio, $\frac{d}{d_o}$ |
| Density ratio, $\frac{\rho}{\rho_o}$ | Water depth ratio, $\frac{d}{d_o}$ |
| Pressure ratio, $\frac{p}{p_o}$ | Square of water depth ratio, $\left(\frac{d}{d_o}\right)^2$ |
| Velocity of sound, $a$ | Wave velocity, $c$ |
| Mach number, $V/a$ | Froude number, $\sqrt{gd}$ |
| Subsonic Flow | Streaming water |
| Supersonic Flow | Shooting water |
| Shock Wave | Hydraulic jump |

Henceforth, unless stated otherwise, Mach number will refer to both $V/a$ and $V/\sqrt{gd}$ , Froude number.

Substituting equation (3a) into the expression

$$M_s = \frac{V_s}{\sqrt{g d_s}}$$

one gets

$$M_s = \left[\frac{2(d_o - d_s)}{d_s}\right]^{\frac{1}{2}} \tag{13a}$$

Similarly the local Mach number may be found to be

$$M_\ell = \left[\frac{2(d_o - d_\ell)}{d_\ell}\right]^{\frac{1}{2}} \tag{13b}$$

For isentropic channel flow the pressure coefficient may be established as

$$C_p = \frac{p_\ell - p_s}{q_s} = \frac{p_\ell - p_s}{p_s} \frac{2}{\gamma M_s^2} \tag{14}$$

This simplifies, for the hydraulic analogy, to

$$C_p = \left[ \left( \frac{d_l}{d_s} \right)^2 - 1 \right] \frac{1}{M_s^2} \tag{15}$$

# CHAPTER IV

## EQUIPMENT AND PROCEDURE

The water channel.--There are two types of water channels suitable for use in the application of the hydraulic analogy, viz:

(1) One in which the model is stationary and the fluid flows past the model.

(2) One in which the model is moved through still fluid.

Both types are in use today. The Massachusetts Institute of Technology and the NACA at Langley Field use Type 1. Georgia Tech and the University of California have channels of Type 2. The advantages of each may be summarized as follows:

Type 1

(a) Pressure and depth measurements are easily taken by fixed or semi-fixed devices.

(b) Flow patterns may be observed and photographed with greater ease.

(c) Model wear and distortion, especially in the case of wooden models, is negligible and installation is easier.

Type 2

(a) Simplicity and low cost of construction is of prime importance.

(b) A greater range of speeds is possible.

(c) The study of accelerated flow is possible.

(d) There is no boundary layer buildup on channel floor and sides.

(e) The method of establishing speeds is easier.

As stated previously, Type 2, wherein the model is moved through still fluid, is used in this report. A general view of the water channel is shown in Fig. 2, page 46 . The frame is of bolted, structural steel. This frame supports a channel four feet wide, twenty feet long and approximately one and one-fourth inches in depth. The bottom of the channel is one-fourth inch plate glass in two five foot sections and one ten foot section. The ten foot section is located at the "test section" giving a longer span of continuous floor. These plates are supported transversely by steel members at intervals of thirty inches and longitudinally at the edges by the frame. The legs of the frame are at five foot intervals and rest on screw jacks enabling the glass bottom to be leveled to within 0.005 inch at all points. Slight warping of the structure and sag of the glass prevents any closer tolerances to be held with any amount of practicability. The leveling is accomplished by means of a depth gage; this instrument also being used to determine the static depth of the water before each run. This gage can be read to 0.0005 inch and be used with accuracy to measure ± 0.001 inch.

A drain is provided at one end of the channel. The fluid used is common tap water, a supply being located near the drain.

<u>The carriage or model mount.</u>--The movable model carriage is of welded steel tubing to which an appropriate model mount is attached. This mount is adjustable, allowing the model angle of attack to be varied. The present mount, while adequate, should be improved upon, to allow

greater accuracy and convenience of adjustment. The present accuracy is approximately ± 0.5 degrees. The model is allowed to press firmly against the bottom, preventing flow underneath the model. The model carriage moves along the channel sides, which act as rails, on eight rubber wheels. Four of the wheels, one at each corner, carry the carriage weight to the frame. The other four are situated to prevent lateral motion. The inset in Fig. 2 illustrates the carriage with model in position as used in this work.

Carriage drive.--A one-quarter horsepower, single phase, alternating current electric motor is used to drive the carriage. This reversible motor uses pulleys and a 3/32 inch, continuous aircraft cable of multi-strand steel to tow the carriage. The carriage motion control is obtained by a reversing mechanism and a "Speed-Ranger" device. This gives control in either direction for constant speed runs. For varying speeds or accelerating runs an auxiliary power supply is used. This system consists of a 24 volt, 14.5 ampere direct current, series wound motor which drives the carriage cable through a set of reduction gears.

By use of the above arrangement, steady speeds of from 0.5 to 5.5 feet per second may be realized with ease. The speeds at the beginning and end of a run never varied more than 0.5 per cent. Only slight vibration attributable to the drive mechanism was noticed.

A means of accurately timing the speed of the model is made possible by the use of a microswitch placed on the track. A cam 2.925 feet in length is attached to the carriage. This cam trips the switch which automatically operates a timer located on the control panel. This timer can be read to 0.1 of a milli-minute and the accuracy is within this limit. The control panel containing the timer also houses switches for starting, reversing, operating the drive mechanism, and photo flood lights. A pistol grip hand switch on a long extension cord was constructed so that the drive could be started while the operator was removed from the control panel. This feature was necessary because of the method used in measuring the water depth around the model.

Depth measurement.--The first experimental data taken from the Georgia Tech water channel measured the water depth distribution around a model by photographic means. The method and equipment used are described by Hatch, Ref. 14. Catchpole, in Ref. 15 also used this method and suggested an improved measuring technique by the use of probes. Visual observation was used to determine when the probes first touched the water. This was found to be subject to considerable error so another method was developed.

This method consisted of insulating the probe from the water then lowering the probe until contact with the water is made. This completes the grid circuit of a vacuum tube, causing a relay to operate a signal light. Results obtained with this technique have been shown to be closer to theoretical and experimental results than the other methods. This probe method was used in all experiments in this report.

When the probe method is used, a plexiglass bracket is mounted firmly on top of the model. Probes are made by mounting a steel needle in a brass screw of suitable length. Tapped aluminum bushings are pressed in holes drilled in the plexiglass and the probes screwed into these bushings. Photographs of a model with probes is shown in Fig. 3. The holes being so drilled to place the probe points approximately one-eighth inch from the model, thereby measuring the water depth at this distance from the model. This distance had been suggested in earlier works. Irregularities in drilling the holes in the plexiglass and needles off center or bent due to handling caused the tips to spiral as the probes were lowered by screwing into the bushing. This effect was decreased to a minimum by plexiglass spacers glued to the model between model and probe. The distance of probe tip to model was measured after most runs and found to be fairly constant for individual probes. A tapped wood cover was used to take out play in the tapped bushings. Copper or aluminum contacts were used in conjunction with a spring clip to connect the probes to the grid circuit.

A meniscus of approximately one-fourth inch in height on model and starting three-eights inch from the model was observed. Since the probe would also be measuring meniscus height, measurements were also made in static water of known depth, with the model still, to ascertain the error due to the meniscus. It is assumed that this meniscus varies little with speed and static depth error is directly applicable to the measurements around the moving model. Fig. 4 illustrates the problem posed by the existance of the meniscus.

Models.--The airfoils tested were the British Goldstein 1442/1547 "roof-top section" and the Gottingen 624. The Goldstein section was tested in the flexible wall N. P. L. tunnel by Pearcey and Beavan, Ref. 16, at Mach numbers up to 0.87 giving a comparison of water-channel results at subsonic and low transonic Mach numbers. The design conditions for this section are a $c_l = 0.2$ at $\alpha = 0.5$ degrees, with flow velocity at 0.6 chord (upper surface) reaching the speed of sound at a free stream Mach number of 0.694. It has a maximum thickness of 14.0 per cent at 0.42C and a camber (American type, a = 0.6) of 1.5 per cent at 0.47C.

The Gottingen 624 is similar to familiar Clark Y sections in that it has a flat under surface aft of 0.3C. It is 16 per cent thick at 0.3C with a finite thickness trailing edge. Results of wind-tunnel tests at Guidonia at a Mach number of 1.47 are given in Ref. 17. The results of these tests give a comparison at the upper limits of the transonic region. Fig. 5 shows airfoil ordinates and probe positions.

The models were constructed of laminated mahogany with several coats of shellac for waterproofing. Both were of 24.0 inch chord. Wood was chosen because of its light weight.

<u>Additional equipment</u>.--A length of brass rod was laid in the channel and connected to the ground of the indicator light relay circuit providing a mutual ground connection. Salt was sprinkled in the channel to insure ionization and therefore better conductivity of the water.

Since interference from reflected shock waves was noted at certain speeds, ordinary fine mesh wire was used to line the sides of the test section. These screens absorbed and dissipated the hydraulic jump waves so that they were not reflected from the channel sides back to the model. This is illustrated in Fig. 6 and proved very effective.

<u>Testing procedure</u>.--In preceeding experiments using the probe method the probe was lowered to an approximated mean position. Then the run was made to see if the probe remained in the water or if the depth at that point was lower than that of the probe tip. These runs were repeated until either the probe just touched the water in the case of depth greater than static. In the case of depth less than static the runs were repeated until a fraction of a turn would either make or break contact with the water. This method is slow, and in the case of local depth less than free stream, inaccurate. It was found in both static and moving tests that the water adhered to the probe after immersion and an error as great or greater than 0.020 inch could be incurred due to this surface tension. To overcome this inaccuracy a slight modification of the probe method was used. In this method the operator moved with the model lowering the probe until contact was made. The extension swith described previously was used to enable this control of the movement while adjusting the probes.

The probes are lowered from rear to front one at a time to prevent interference caused by probe wake. When all probes have been adjusted the model is removed from the channel. The local depth of the water is then measured using a surface plate and height gage. The accuracy of the gage is 0.001 inch.

Preceeding each test run the average static water depth of the test section and immediate area was adjusted. Depths were held to ± 0.001 inch. Carriage speed was then adjusted until the timer read the desired calculated time within ± 0.2 milliminute. The model was wetted each time to insure equal meniscus effect. The passes were then made, allowing time in between starts to allow the water to settle until no motion was visible. The probes were adjusted and the depths measured and connections made as stated in the previous section.

The angle of attack of the model was set by using the rotable, calibrated model mount.

<u>Tests conducted</u>.--Both airfoils were tested under the following conditions:

| Mach Number (Froude No.) | Static Water Depth, inches |
|---|---|
| 0.57 | 0.500 |
| 0.67 | 0.500 |
| 0.85 | 0.500 |
| 0.85 | 0.250 |
| 1.00 | 0.250 |
| 1.20 | 0.250 |
| 1.47 | 0.250 |

The following angles of attack were used at each Mach number:

-2.5, -1.5, 0.5, 2.5, 4.5, 6.5 degrees

The greater depth was used at the lower Mach numbers to increase the mechanical accuracy of the depth measurements.

<u>Determination of coefficients</u>.--The speed of the model was calculated for a given Mach number, by the relationship $v_m = M\sqrt{g\, d_s}$ and equation (13b) was used to determine $d_o$ .

After obtaining the local depth variation the pressure coefficient was determined using equation (14). The pressure coefficients were then plotted versus per cent chord and per cent thickness. These plots, when integrated by a planimeter (area and first moment) yield

$C_N$ , $C_c$ and $c_{m_{c/4}}$ . The section lift and drag coefficients were then calculated from the following relationships.

$$c_l = C_N \cos\alpha - C_c \sin\alpha$$

$$c_d = C_c \cos\alpha + C_N \sin\alpha$$

# CHAPTER V

## DISCUSSION OF RESULTS

This thesis was initiated to investigate the application of the hydraulic analogy to testing in the transonic region. The data obtained in the above investigation was sufficient to enable a comparison of two airfoils operating in this region. The results and related factors of each problem will be discussed in turn.

### The Hydraulic Analogy

Evaluating water channel results by comparison with existing data leads immediately to complications. The only data available for this purpose is meager wind-tunnel data. Theory is noticably scarce concerning airfoils of this type in the transonic region. Therefore in evaluating the analogy, not only must factors affecting the results of the water channel be examined, but also the validity of the water channel-wind-tunnel comparison must be questioned.

Factors affecting the accuracy of the analogy will be discussed first.

Determination of test mach number.--The assumption that the wave velocity

$$c = \sqrt{g\, d_s}$$

is not entirely correct. As Laitone, Ref. 13, and others have pointed out, the "group velocity" determines the equivalent Mach number in the analogy. If the free surface flow is composed of waves of different lengths, the group velocity will be determined, in part, by the shortest wave length present. In Ref. 18, Milne-Thompson shows the group velocity to be

$$c_{group} = c_{\lambda=\infty} - \lambda_s \frac{dc}{d\lambda}$$

where $c_{\lambda=\infty} = \sqrt{g\,d_s}$

$\lambda_s$ = shortest wave length

$\frac{dc}{d\lambda}$ = change of celerity with respect to wave length

Fig. 7 illustrates the variation of wave velocity with wave length. The effect of surface tension and depth is clearly shown. The exact equation for wave velocity is

$$c = \sqrt{\left(\frac{g\,d_s}{2\pi} + \frac{2\pi T}{\lambda\rho}\right) \tanh\left(\frac{2\pi d_s}{\lambda}\right)}$$

where T = surface tension

When $d_s \ll \lambda$ and $T \ll \lambda\rho$, the above equation reduces to

$$c = \sqrt{g\,d_s}$$

The figure shows the necessity of a large model and water depth at a practical minimum. However, even if the model size is relatively large, the disturbance may be partially composed of small wave lengths. This is especially true when depth varies rapidly along the model surface. The effect of the reduced wave length would be to lower the disturbance propogation rate (group velocity). This, in turn, causes the ratio (model velocity)/(disturbance velocity), i.e. Mach number, to be higher than that indicated by the analogy where model velocity and $\sqrt{g d_s}$ determine the Mach number. Fig. 7 shows that a depth of 0.20 inches is desirable if tap water (surface tension = .00416 lb./in.) is used as the fluid.

Wall interference.--No attempt was made to determine the effect of the channel boundaries on the pressures at the model. It would appear that such would exist, especially at the higher Mach numbers where it is visually apparent that the pressure variation extends to the wall. The author believes that an arrangement similar to that used to minimize shock reflection could be used to minimize boundary effects.

Blocking or chocking.--Blocking or chocking m ans that a sonic line from model to wall is established. When this occurs in the wind-tunnel further attempts to increase flow veloc ities past this Mach number result in increased power and stagnation pressure, but no increase in Mach number. The sonic line will remain approximately at the minimum area point until a normal shock "pops" in front of the model. The model will then be operating in subsonic conditions behind the normal shock in the tunnel.

This important Mach number can be computed on an area basis. M = 0.87 was greater than the calculated blocking speed by either one-dimensional flow theory or computations based on the methods in Ref. 19.

However, the aforementioned shock wave was not noted as the model traveled down the channel at any of the test Mach numbers. An exception to the last statement would be the test at a Mach number of 1.00 where the shock would be expected to appear.

Two main problems complicate the comparison of water channel results to wind-tunnel data. The first is doubly important since it is necessary to establish the analogy.

$\gamma$ = 2.0.--The basic requirement for the analogy is that $\gamma = 2.0$ for the analogous gas flow. Therefore water channel results can compare directly with wind-tunnel results only if the effect of varying $\gamma$ is small. It is necessary to examine the significance of this factor in gas flow.

When considering two-dimensional airfoil tests, pressure distribution is of prime importance. In incompressible flow this pressure will be a function of the square of the (local velocity)/(free stream velocity) ratio. Similarity of flow velocity is equivalent to similarity of pressure distribution. However, compressible flow presents another problem. As seen in the equation

$$\frac{\gamma-1}{2} M_l^2 + 1 = \left(\frac{p_o}{p_l}\right)^{\frac{\gamma-1}{\gamma}}$$

pressure distribution is no longer solely dependent on local velocity (Mach number) but also on $\gamma$ . Therefore, similarity of both pressure distribution and local Mach number can not exist simultaneously when comparing flows of different specific heat ratios. This is demonstrated in Fig. 8. It is to be noted that, as Mach number increases supersonically the effect of $\gamma$ decreases. Also significant is the fact that a lower critical pressure ratio (where M = 1.0) is required when $\gamma = 2.0$. The pressure coefficient is also affected by $\gamma$ as seen in equation (14). The similarity rules, mentioned on page 9, point out the significance of $\gamma$ in transonic flow.

Various methods of modifying the analogy are suggested by the flow equations for gases and free surface liquids. One such modification, suggested by Ippen and Harleman in Ref. 14, considers the continuity equations. Here the relationship

$$\frac{d}{d_o} = \frac{\rho}{\rho_o} \qquad \text{Equation (7)}$$

is not dependent on $\gamma$. Pressure coefficients are then determined using $\gamma = 1.4$. The effect of this modification is shown in Figs. 9 and 11. Changes in pressure distribution using this modification are shown in Fig. 9 for two typical check points. There is a marked change at M = 1.47, however little difference is evident at the subsonic Mach number. Lift and drag coefficients are therefore relatively unaffected at the lower test velocities. The effect at the higher Mach number is adverse as for as correlation with wind-tunnel data is concerned. Since the modification shows no increase in correlation, no attempt was made to incorporate the modification in the airfoil comparison. The assumptions of isentropic flow are invalid for flow with a discontiunity. However, using normal shock conditions when shock waves occured did not affect the subsonic data. The effect at M = 1.47 was about half that of $\frac{\rho}{\rho_o}$ modification. Since the shock is detached and curved other methods should be used to correct for entropy increase.

Viscous effects.--In transonic flow Reynolds number is of considerable importance. Not only is laminar or turbulent flow a governing factor in pressure distribution preceeding the shock wave but the shape of the shock wave is determined in part by the type of boundary layer. The shock wave-boundary layer interaction also affects the pressure distribution following the shock wave. Fig. 10 illustrates this fact. This figure, taken from Ref. 5, shows shock wave and pressure distribution variation with Reynolds number for the same circular are airfoil. Reynolds numbers were not listed for the Goldstein airfoil tests but the illustrations of shock waves suggest laminar flow. The $\lambda$ shock, characteristic of laminar flow, was observed in their tests. In the water channel all observed shock waves were more of the type associated with turbulent flow. However, at high supersonic Mach numbers the viscous effects are reduced considerably, especially in the case of attached shocks.

Comparison with wind-tunnel results.--It has been shown in the previous discussion that several items may affect a direct comparison of water channel results to those of the wind-tunnel. The data obtained in the experimental work in this thesis does not enable the various effects to be evaluated separately. However, a comparison of the results can determine the possible magnitude and type of error, if such discrepancies exist.

Fig. 11 shows results from wind-tunnel tests at a Mach number of 1.47, Ref. 17. Although scatter is evident, very close comparison is obtained with the direct analogy at this Mach number. This has been true of previous tests of this type conducted at the Georgia Institute of Technology, specifically in Refs. 15 and 20. These tests were, in the main, at supersonic Mach numbers and all used pointed slender airfoils. The previous discussion concerning the effect of $\gamma$ , viscosity and surface tension suggest that the higher the Mach number, the better the comparison with wind-tunnel results.

At this point it would be well to describe the method used in fairing the various curves of lift, drag and moment coefficients. These coefficients were plotted versus Mach number at constant angle of attack and versus angle of attack for constant Mach number. Fig. 12 illustrates this method for the lift coefficients of the Goldstein airfoil. Once these plots are consistent, the data is cross plotted, i.e. lift versus drag and moment in the familiar fashion. This enables the experimenter to establish trends and obtain consistent results even though "scatter" may be in evidence.

Fig. 12 also shows the results of the subsonic wind-tunnel tests for the Goldstein airfoil. It is evident that more water channel data is needed in the subsonic region from $M = 0.4$ to $0.8$. Regardless of the fairing of the curves, it can be seen that there is a discrepancy between the wind-tunnel and water channel data. Evident is the less abrupt decrease in lift (at constant angle of attack) for the water channel results. Also evident is the fact that the peak values are occuring at a lower Mach number. This suggests that compressibility effects are occuring at lower Mach numbers.

The fact that the local speed of sound is reached at a lower pressure coefficient when $\gamma = 2.0$ (or conversely, that for a given pressure coefficient the Mach number is greater for $\gamma = 2.0$) would tend to cause compressibility effects to occur at a lower free stream velocity. This assumes that the pressure distributions would agree until compressibility effects occur.

If group velocity were lower than the celerity used to determine Mach number, the airfoil would actually be in what would correspond to a higher Mach number flow. This was discussed previously under "Wave Velocity".

The fact that the airfoil appears to be at a higher Mach number than that computed is substantiated in Fig. 13. The pressure distribution at Mach 0.57 is in fair agreement, although the pressure distribution obtained in the water channel shows the characteristics of a higher free stream speed than the wind-tunnel data. The lower illustration, however, shows clearly that this mismatch of Mach numbers exists. In this plot the water channel data at $M = 0.67$ agrees very closely with wind-tunnel results obtained at $M = 0.779$. The typical roof-top pressure distribution typical of laminar flow airfoils is evident for the Goldstein at a wind-tunnel Mach number of 0.682, but not in the water channel. On the lower surface, where local speeds are lower, agreement between the near equal Mach numbers is much better. Fig. 14 illustrates a similar comparison of pressure coefficients. This data, plotted from Ref. 12, also shows water channel results in closer agreement at the high negative pressure coefficiency, with wind-tunnel data obtained at a higher Mach number than that obtained at the same speed.

It is likely that surface tension, $\gamma$ effects and viscosity differences all contribute to these discrepancies. As was previously mentioned, it is impossible to evaluate these effects separately from the data contained in this thesis. Fig. 15 shows the effect of water depth, the smaller depth perhaps being less liable to error from surface tension phenomena. Although agreement with wind-tunnel pressures is poor the shock wave position (evidenced by the peak pressure and rapid fall-off) is further aft for the case of the greater free stream depth. This is true also in Figs. 24 and 25. However, the lift coefficient for this depth is almost identical to that of the wind-tunnel at this speed and angle of attack. Observed shock waves in the water channel normally occured further aft and at lower Mach numbers than in the wind-tunnel.

On the basis of the data discussed above, it appears that high subsonic water channel data is qualitative at best, at least for this type of airfoil. It also appears that, as test speeds increase supersonically, the data from the water channel approaches that of the wind-tunnel. It is evident that further experimentation is desirable to determine the various phenomena that tend to cause error between the hydraulic analogy and wind-tunnel tests.

## Airfoil Comparison

It is apparent from the preceeding discussion that this data cannot be compared quantitatively with wind-tunnel results. Since the Goldstein airfoil is designed for viscous flow, there may be some question concerning the validity of a qualitative comparison of airfoils in the subsonic region because the effects of viscosity may differ in the two testing methods. If pressure distribution affects wave length to the extent that group velocity is altered, the airfoils may be operating at different Mach numbers, although in the direct analogy they would be equal. However, the relative level of various parameters would not be affected by the existence of such discrepancies.

Plots of the lift curve, drag polars and pitching moments are presented in Figs. 16 through 19. Here, in standard form, are the results of the water channel at all Mach numbers. However, certain parameters that define these curves are more useful and present data in condensed form. These various parameters are shown in Figs. 21, 22 and 23 plotted versus Mach number to show the effect of the airfoil shape and the variation with Mach number.

These parameters define the lift curve, drag polars and moment curves. The word define must be used in a limited sense however. For example, drag polars are not exactly parabolic, lift and moment curves are not necessarily straight lines. However, normally of main interest is that range of lift coefficients where the straight line, parabola, etc., approximates are accurate enough for engineering usage. This is especially true of the speed range that is the concern of this thesis. Fig. 20 illustrates the idealized cases and the defining parameters for these conditions. These parameters are familiar to the aeronautical engineer and need not be discussed in detail. They are presented here to avoid misinterpretation of the data. The drag due to lift factor for a cambered airfoil, K, is not shown, since to accurately define this parameter, a wider range of $c_l$'s is required.

Lift.--Fig. 21 illustrates the various parameters concerning lift. The shaded areas in Figs. 21, 22 and 23 demonstrate the variance from the idealized case for slopes and in scatter due to possible fairing. The lift parameters show that the airfoil shape does not affect, to a great degree the variation of the parameter with Mach number. However, the magnitude of the parameter is affected. As would be expected from examination of the camber of the two airfoils, $\alpha_{LO}$ is more negative for the Göttingen airfoil. However, this effect of camber seems to diminish at higher Mach numbers. The lift coefficient for minimum drag is another function of camber and here, too, the effect of camber decreases with increasing Mach number. The $c_l$ for minimum drag actually goes negative over a considerable range of Mach numbers. It would seem unlikely that this is the true picture, however. The variation of $c_l$ shows the superiority of the more nearly symmetrical airfoil of the higher Mach numbers. This factor for the Goldstein airfoil is approximately fifty per cent higher in this region. The variation of lift with angle of attack at M = 0.57 shows a similarity to shock stall. However, there is not enough data to establish the occurance of this phenomena.

Drag.--Drag parameters are presented in Fig. 22. The drag due to lift and the minimum drags of the Goldstein 1442/1547 are much lower at the higher speeds. This fact demonstrates the importance of the shape of thick airfoils in the transonic region. This difference could be caused by the chordwise position of maximum thickness, the type of thickness variation, or both. Note that the previously mentioned negative $c_\ell$ for minimum $c_d$ causes $c_{d_{PE}}$ to be higher than $c_{d_{min}}$ (Goldstein airfoil). A peculiar effect, evident for both airfoils, is the slight increase in $c_{d_{min}}$ with Mach number following the decrease from the abrupt peak.

Moment.--The aerodynamic center location is of singular importance and is shown rather than $d\,c_{m_{1/4}} / d\,c_\ell$. The shift in a.c. through the difference in location decreases with Mach number. A peculiar note in this situation is that, with increasing Mach number, the aerodynamic center moves forward on the Gottingen airfoil aft on the Goldstein section. The variation of $c_{m_o}$ is markedly different for the two airfoils. However, much of the data for the Gottingen section was obtained by extrapolation and is doubtful in nature.

Pressure distribution.--Figs. 24 and 25 demonstrate the difference in the pressure distribution of the two airfoils at constant angle of attack and various Mach numbers. The particular angles of attack were chosen because at Mach numbers of 0.57 the lift values were nearly equal. The effect of shape at the 0.57 Mach number is demonstrated. Although the lift values are the same, the distribution is quite different. Amount and type of camber account for the near equality of lift at the different angles of attack. It can be seen from a comparison of these pressure distributions that the advantage of the high camber diminishes at the higher Mach numbers. At M = 1.47 the Gottingen airfoil is near zero lift, while the 1447/1547 airfoil is at a $c_\ell$ of near 0.1.

Summary of Discussion

Results obtained from the water channel at the higher Mach numbers are more consistent and are in better agreement with wind-tunnel. Although this could be true of only this type of airfoil, a combination of viscous effects and physical phenomena associated with the application of the analogy could bring about the discrepancies at the lower speeds. However, there seems to be no evident reason why, with caution, the hydraulic analogy cannot be used for comparison purposes where qualitative results are sufficient. The usefulness and ease of application of the analogy to scientific investigation warrants increased study and experimentation. This further investigation is necessary to determine the full extent of the application and limitations of the analogy.

As previously mentioned, the comparison of the airfoils tested is restricted at the high subsonic velocities. Both scatter and detrimental effects of various factors create doubt in this region. The change of the free stream depth at Mach 0.85 affects the observed variation of parameters with Mach number. A qualitative comparison at the lower supersonic velocities shows the Goldstein 1442/1547 to have less minimum drag, greater variation of lift with angle of attack and less drag due to lift. Also evidenced in both airfoils is that camber is less effective at the higher Mach numbers. Although, after an initial decrease, the camber effects of the Goldstein airfoil show a gradual increase. This increase never approached the subsonic level, however.

# CHAPTER VI

## CONCLUSIONS

The results of this group of experiments, although based on a limited amount of data, are sufficient to derive conclusions consistent with the scope of the thesis and the type of experimental data.

1. At higher Mach numbers (wherein subsonic flow is at a minimum) water channel results are in close agreement with wind-tunnel tests.

2. The direct application of the hydraulic analogy does not yield results compatible with wind-tunnel data at high subsonic Mach numbers.

3. The individual, as well as accumulative, effects of surface tension, viscosity, $\gamma$ difference and boundary interference on the hydraulic analogy and the wind-tunnel-water channel comparisons must be evaluated before further testing in the transonic and subsonic region.

4. The airfoil designed for high subsonic operation is superior to the low subsonic airfoil throughout the transonic region. Superior refers to having greater aerodynamic efficiency and other desirable characteristics.

5. The effect of camber on the characteristics of airfoils diminishes as Mach numbers increase supersonically.

6. The hydraulic analogy is a desirable method for evaluating two-dimensional supersonic theories.

# CHAPTER VII

## RECOMMENDATIONS

As a result of the experiments performed preparing this thesis, the author has several recommendations. These suggestions concern equipment improvement, experimental technique and future research.

If many experiments with one model are to be conducted, it is advisable to have metal as that part of the model in contact with the channel bottom and the water. This would prevent the edges from being torn and thereby affecting the original contours of the model. Solid metal models, if not too heavy, are desirable. Otherwise it is recommended that the bottom and a portion of the sides be encased with a suitable, noncorrosive metal. Aluminum is suggested.

While probably the most versatile and easily constructed, the probe method of measurement, as applied in these experiments, has the disadvantage of requiring a large amount of experimental time in the collection of data. It is suggested that a small electric motor with automatic braking device and revolution counter be constructed to lower the probe. Such a device could be constructed to measure depth at any point desired on the model. Although the initial time to design, build and test such a device would be quite large, the time saved thereafter would be considerable. With a quick, accurate method for depth measurement, much more data could be taken, insuring more accurate results.

Several themes for additional research were suggested throughout the experiments and during the analysis of the results. These are enumerated below:

1. Effect of viscosity in the water channel. A study to determine the effect of viscosity on pressure distribution, shock wave type and location and to correlate with Reynolds number and type of boundary layer in viscous gas flow.

2. Effect of surface tension and wave length on model testing. This study should determine if abrupt pressure variations (i.e., depth variation) of the type encountered in blunt models in the subsonic region affects the group velocity and the equivalent Froude number.

3. Blockage effect. A systematic investigation of this phenomena comparing the towed model and the still model with flowing water.

4. Hypersonic testing. A theoretical and experimental study evaluating the application of the hydraulic analogy to hypersonic testing.

5. Theory evaluation. As more and more work is done, both theoretical and experimental, new theories arise. The water channel is a perfect device for the evaluation of such theories is within the limitations of the analogy. A typical example would be the evaluation of Ferrie's method for determining pressure drag behind a detached shock, Ref. 22. Transonic similarity rules are another example.

# APPENDIX

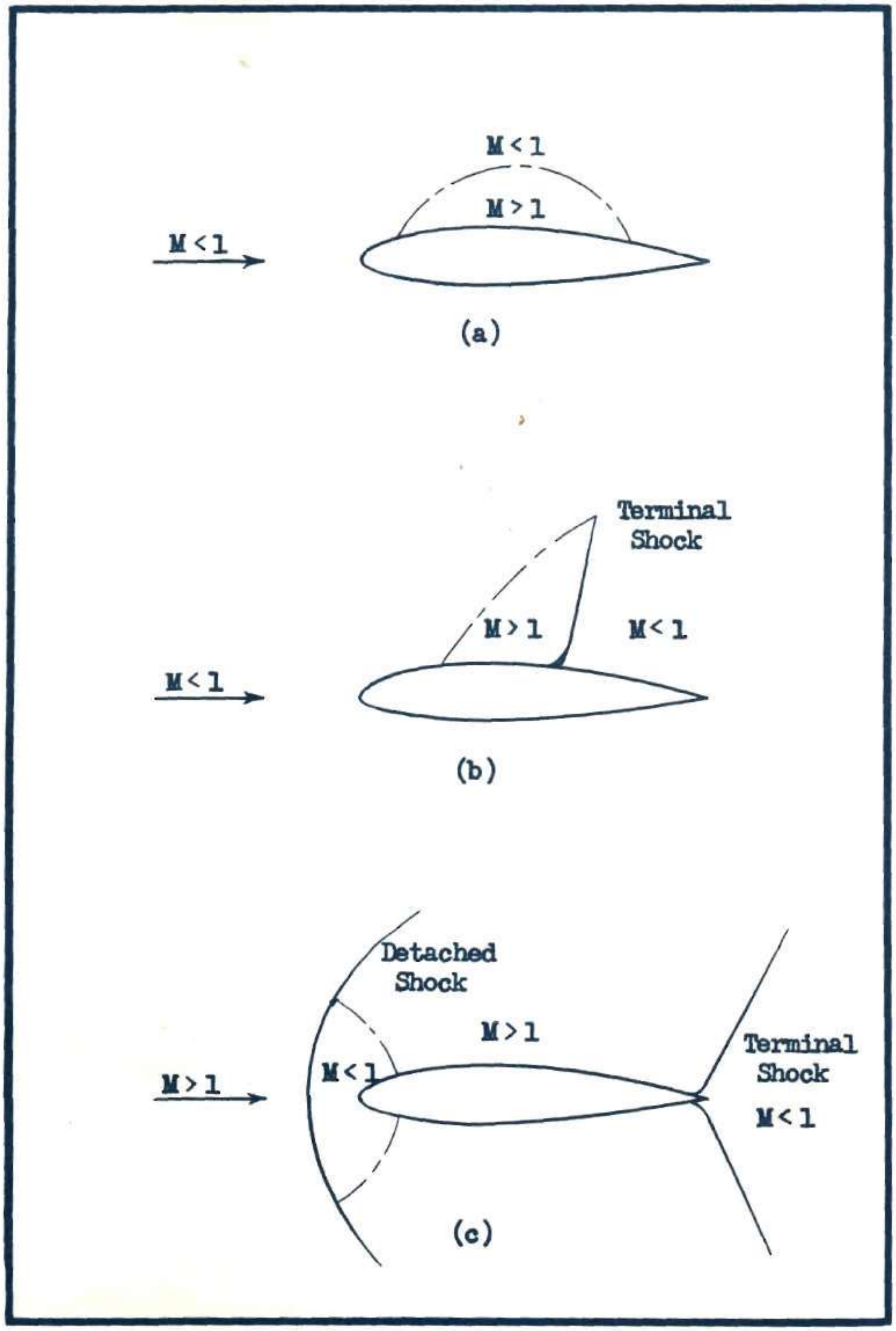



Fig. 1 TYPES OF TRANSONIC FLOW

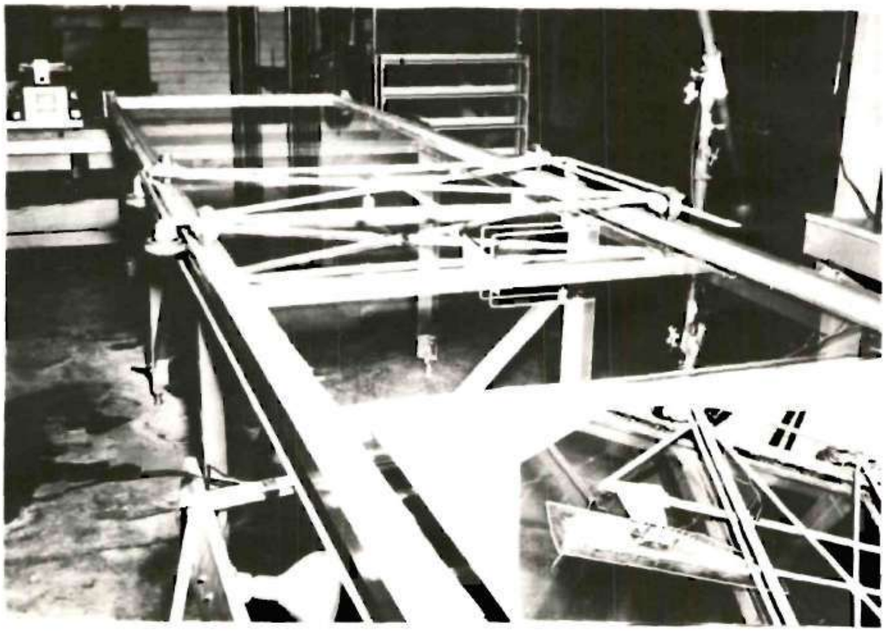


Fig. 2 WATER CHANNEL AND CARRIAGE

Top Side

Fig. 3 MODEL WITH PROBES ATTACHED

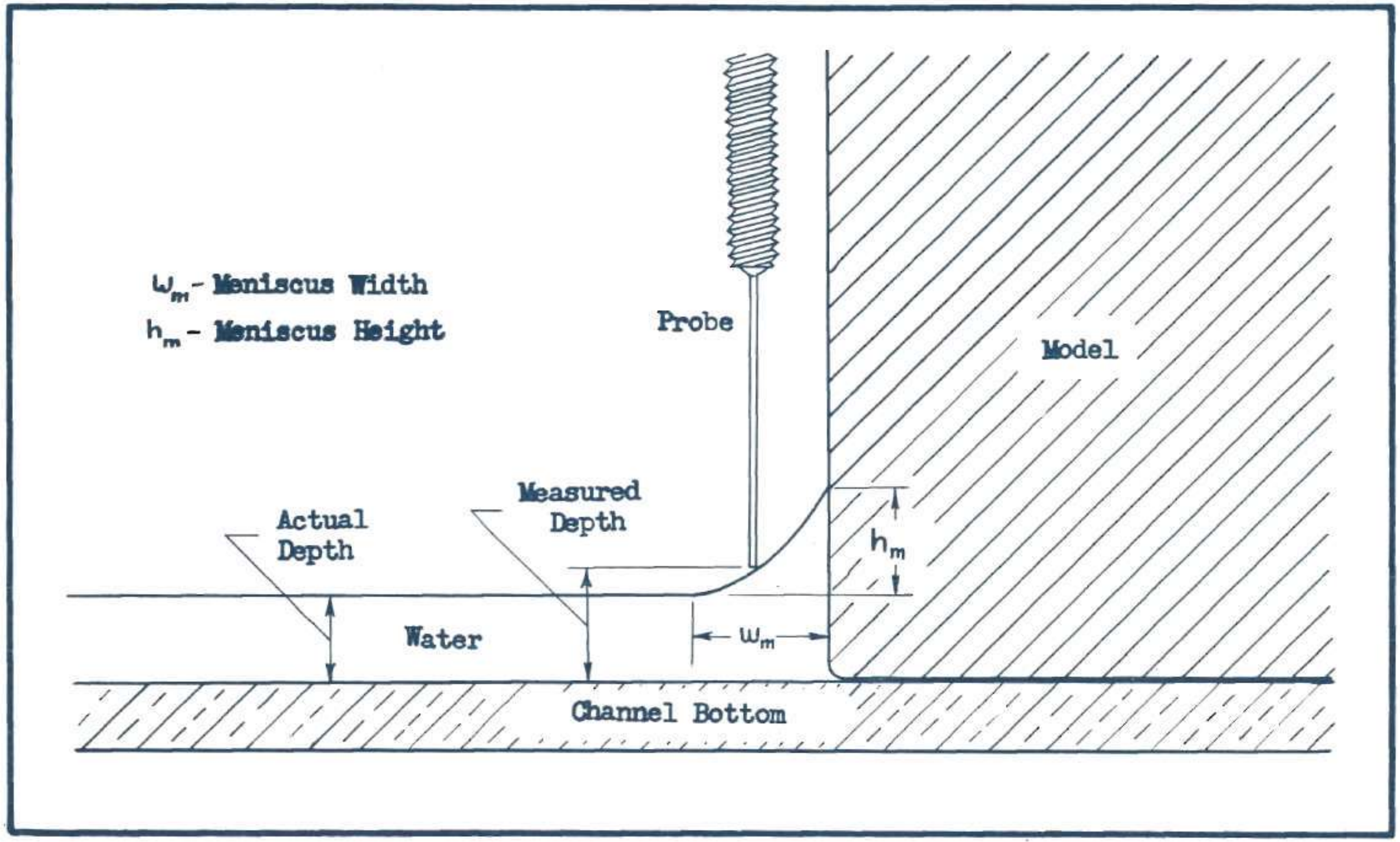
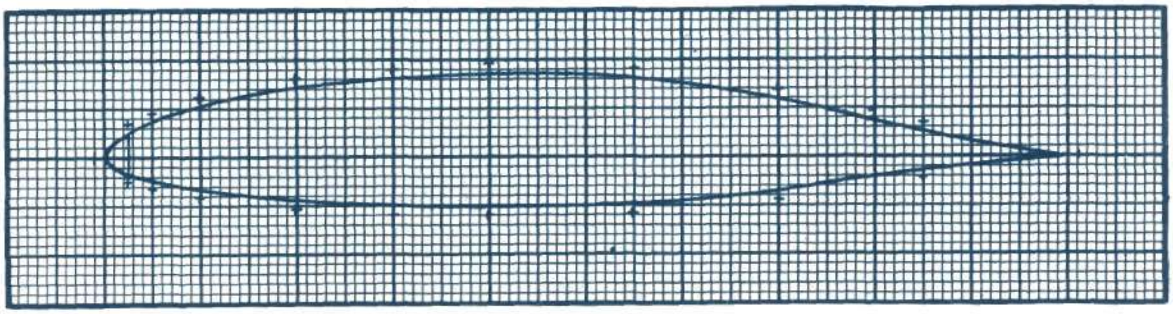


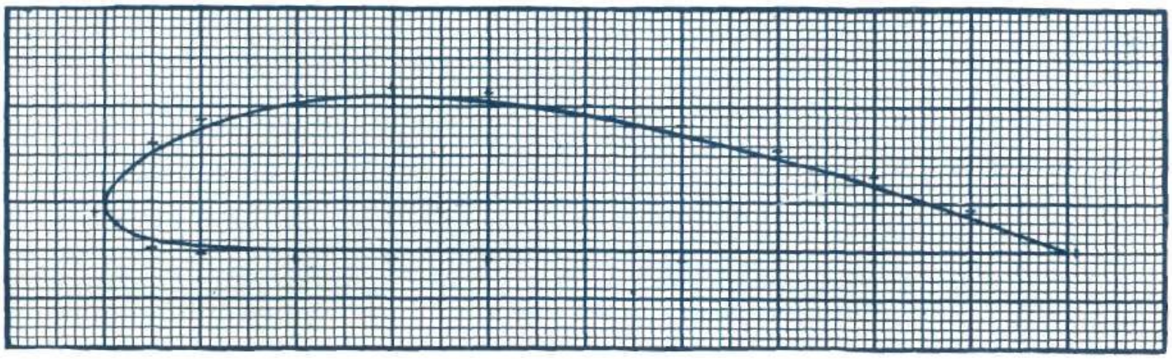

Fig. 4 EFFECT OF MENISCUS ON DEPTH MEASUREMENT

GOLDSTEIN 1442/1547



| $x/c$ | 0 | 2.5 | 5 | 10 | 20 | 30 | 40 | 55 | 70 | 85 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_U/c$ | 0 | 2.60 | 3.71 | 5.20 | 7.02 | 8.03 | 8.45 | 8.04 | 5.81 | 2.60 | 0 |
| $y_L/c$ | 0 | -2.12 | -2.87 | -3.82 | -4.88 | -5.40 | -5.55 | -5.15 | -3.60 | -1.51 | 0 |

GÖTTINGEN 624



| $x/c$ | 0 | 5 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_U/c$ | 4.00 | 10.40 | 12.85 | 15.30 | 16.30 | 15.40 | 14.05 | 12.00 | 9.50 | 6.60 | 3.55 | 0.50 |
| $y_L/c$ | 4.00 | 0.95 | 0.40 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 5 PROBE POSITION AND AIRFOIL ORDINATES

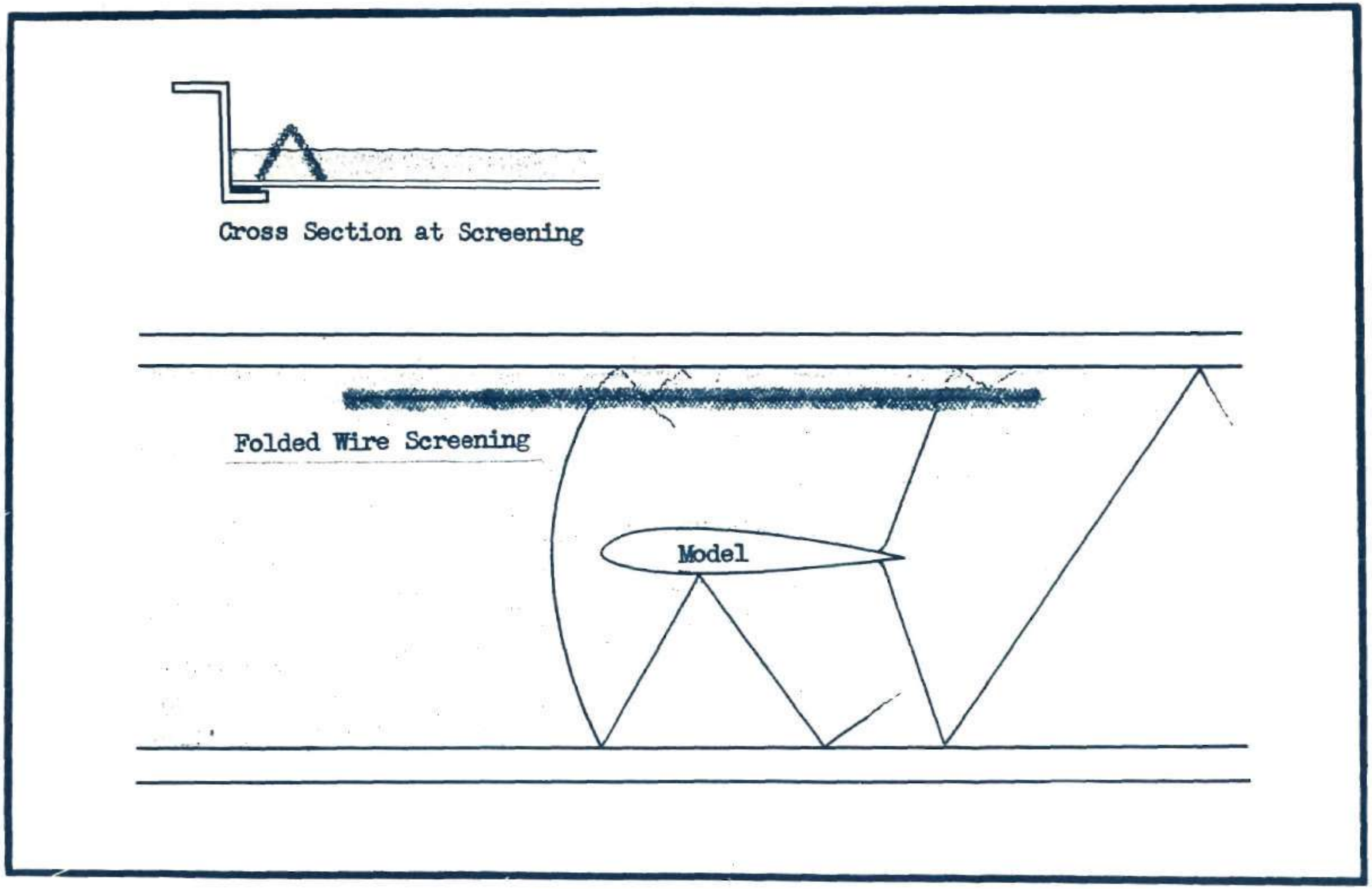



Fig. 6 SHOCK WAVE REFLECTION AND DAMPING SCREENS

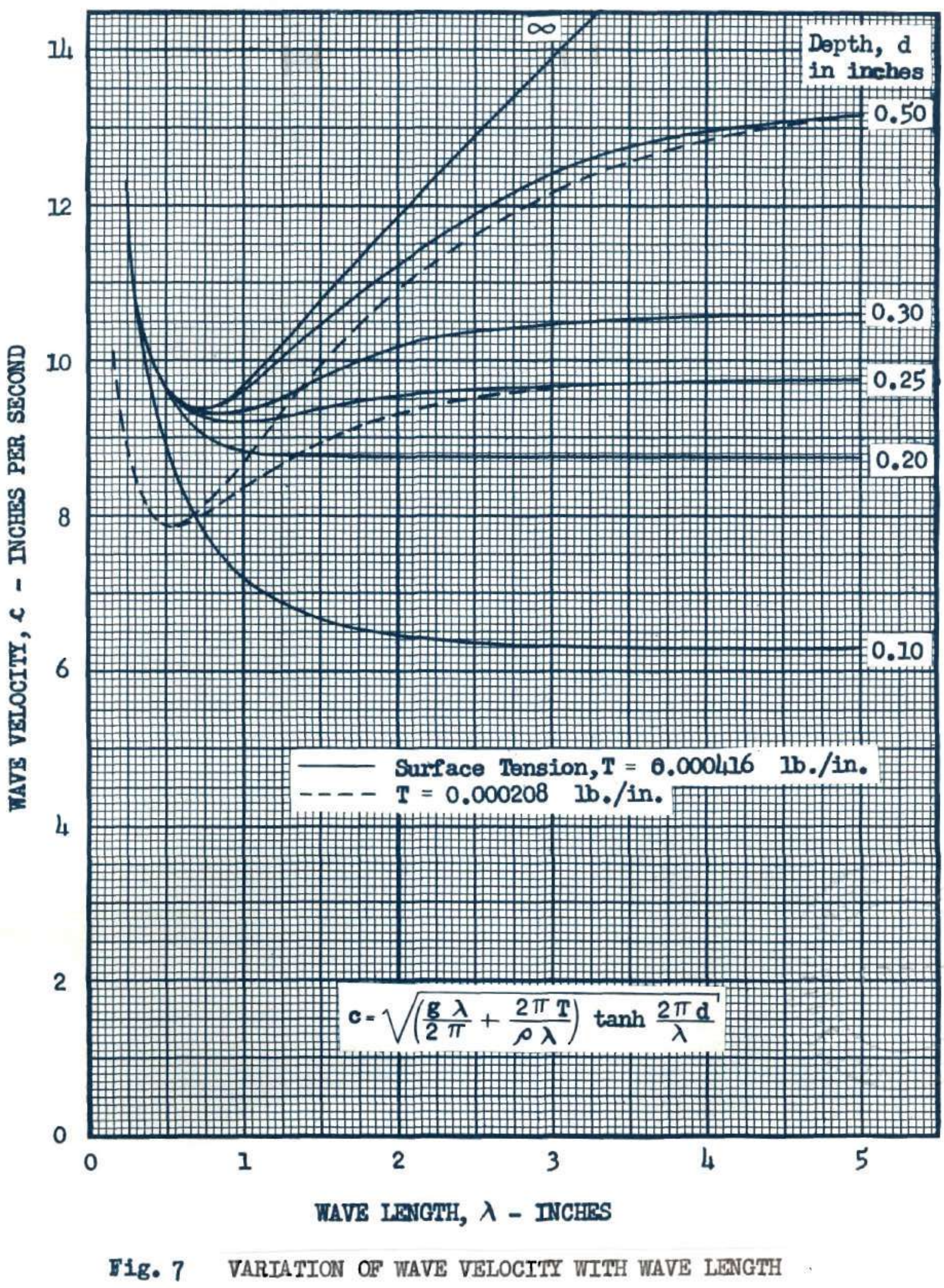



Fig. 7 VARIATION OF WAVE VELOCITY WITH WAVE LENGTH

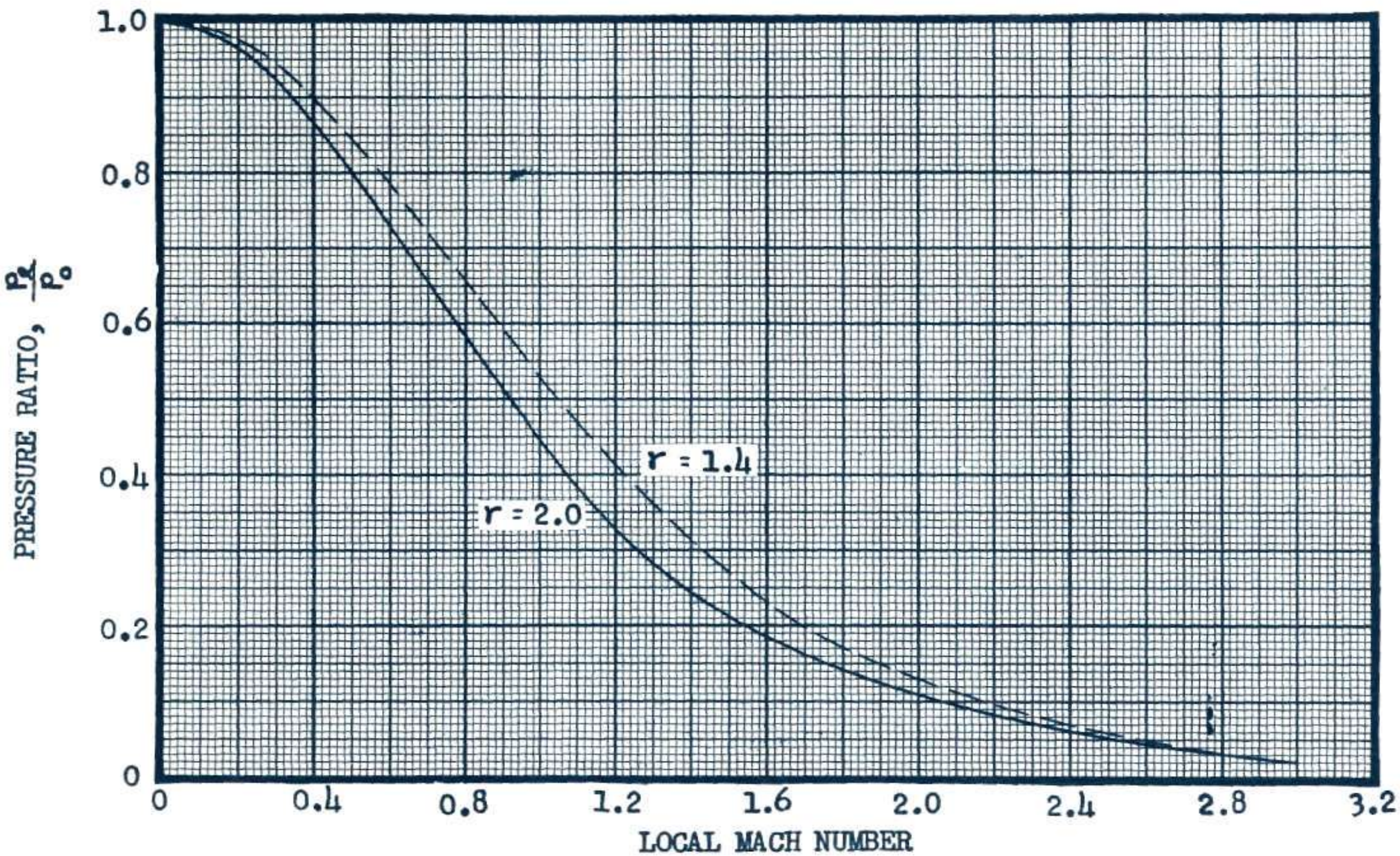
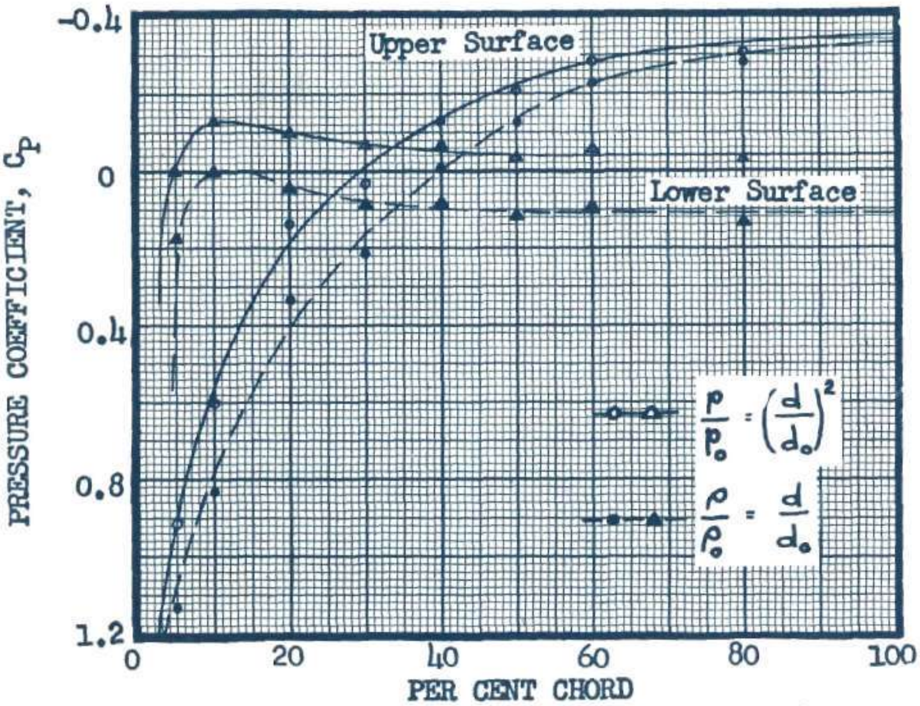
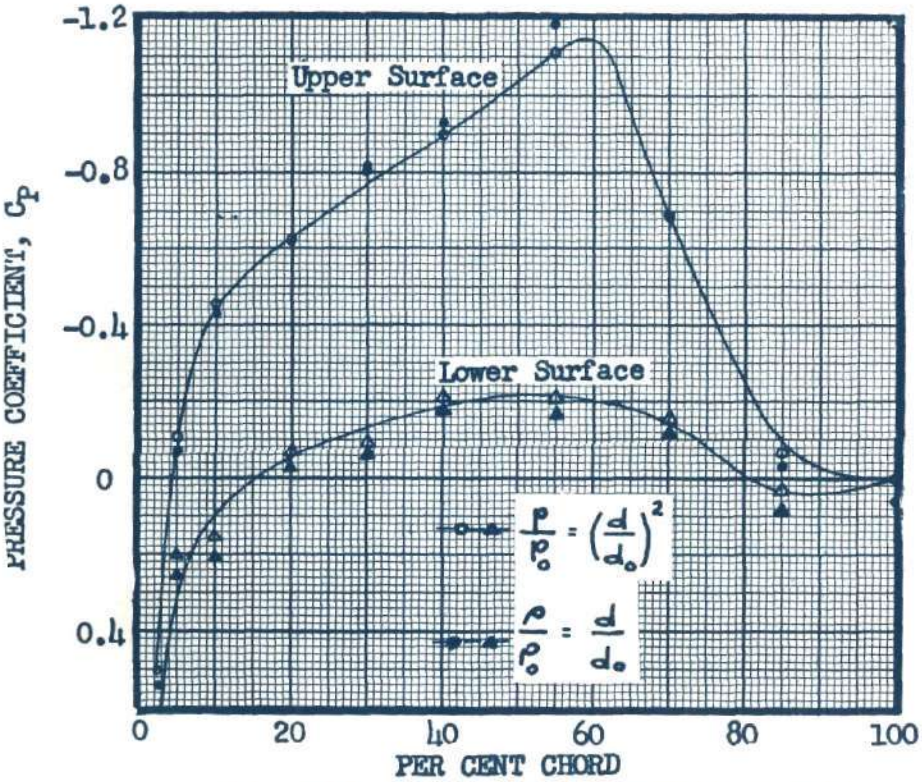



**Fig.** 8 EFFECT OF r AND MACH NUMBER ON PRESSURE RATIO

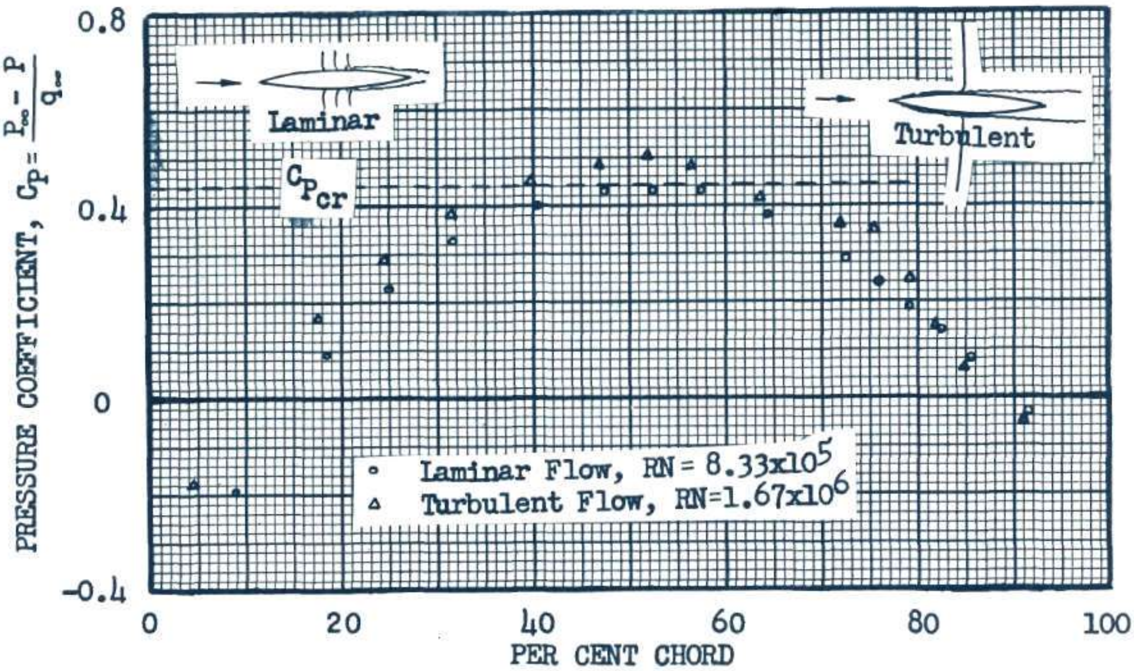



(a) Gottingen 624, M = 1.47, $\alpha$ = -1.5

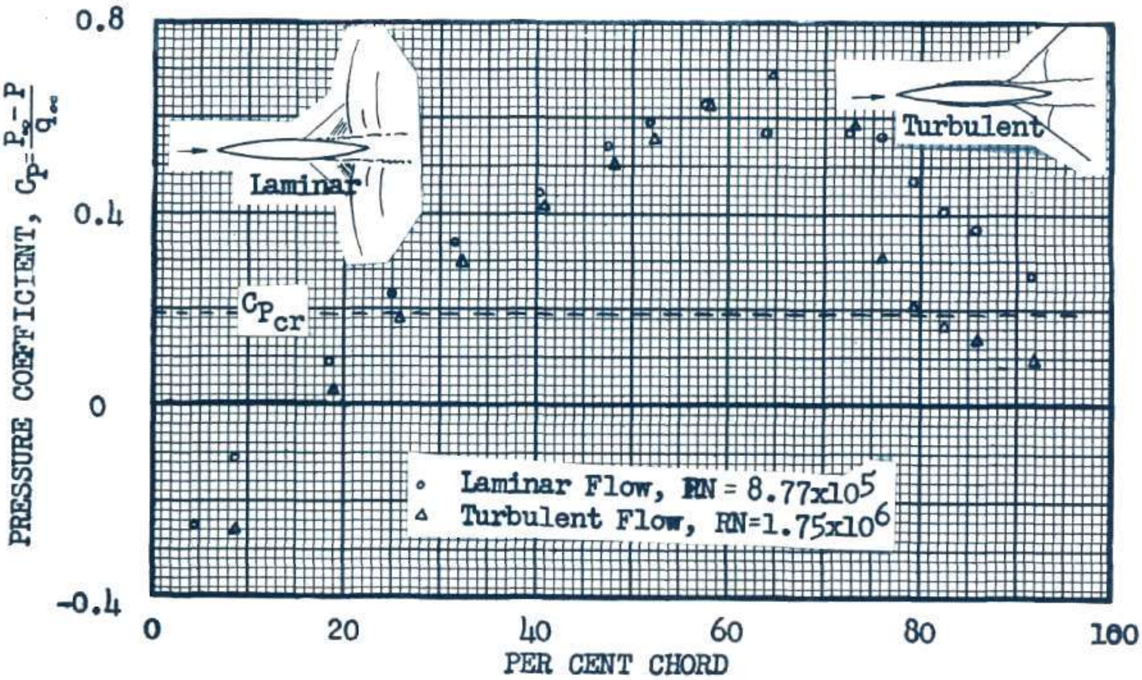



(b) Goldstein 1442/1547, M=0.67, $\alpha$ = 2.5

Fig. 9 EFFECT OF ANALOGY MODIFICATION ON PRESSURE DISTRIBUTION




$M = 0.795 \qquad \alpha = 0^\circ$




$M = 0.895 \qquad \alpha = 0^\circ$

Fig. 10 EFFECT OF TURBULENCE ON PRESSURE DISTRIBUTION AND SHOCK WAVES

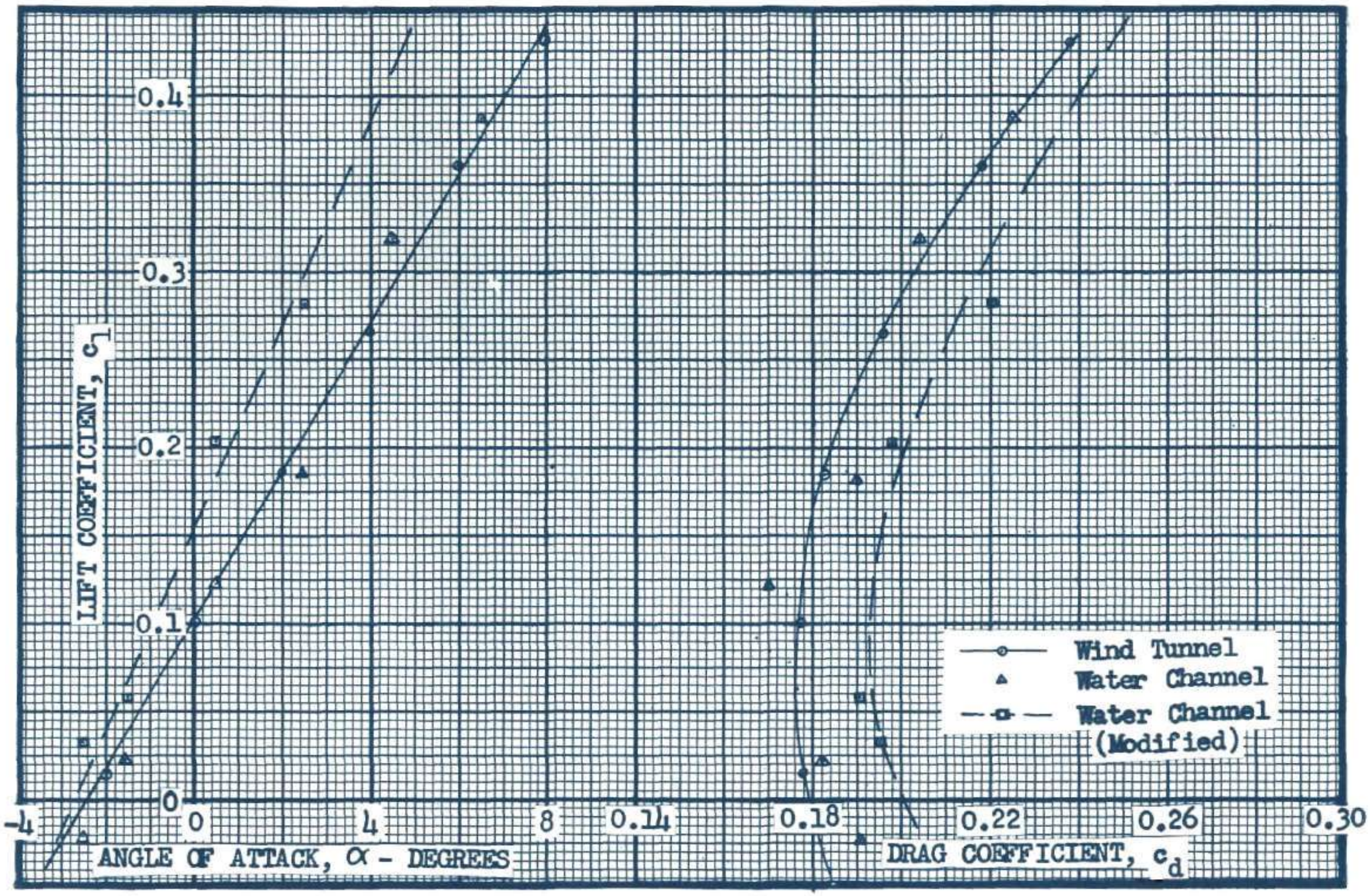



Fig. 11 COMPARISON OF WIND TUNNEL AND WATER CHANNEL RESULTS AT M=1.47 GÖTTINGEN 624 AIRFOIL

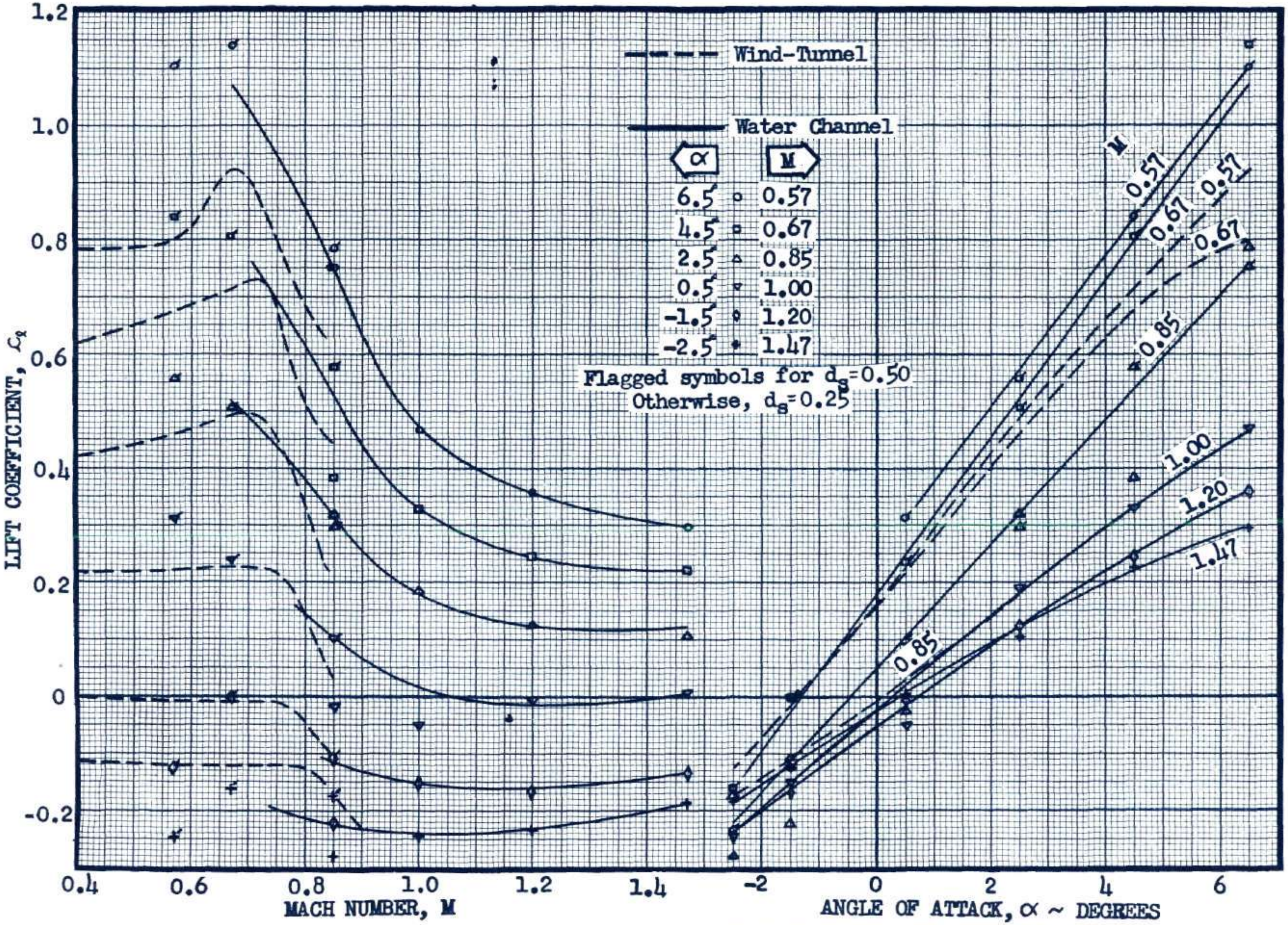



Fig. 12 VARIATION OF LIFT COEFFICIENT WITH MACH NUMBER AND ANGLE OF ATTACK GOLDSTEIN 1442/1547 AIRFOIL

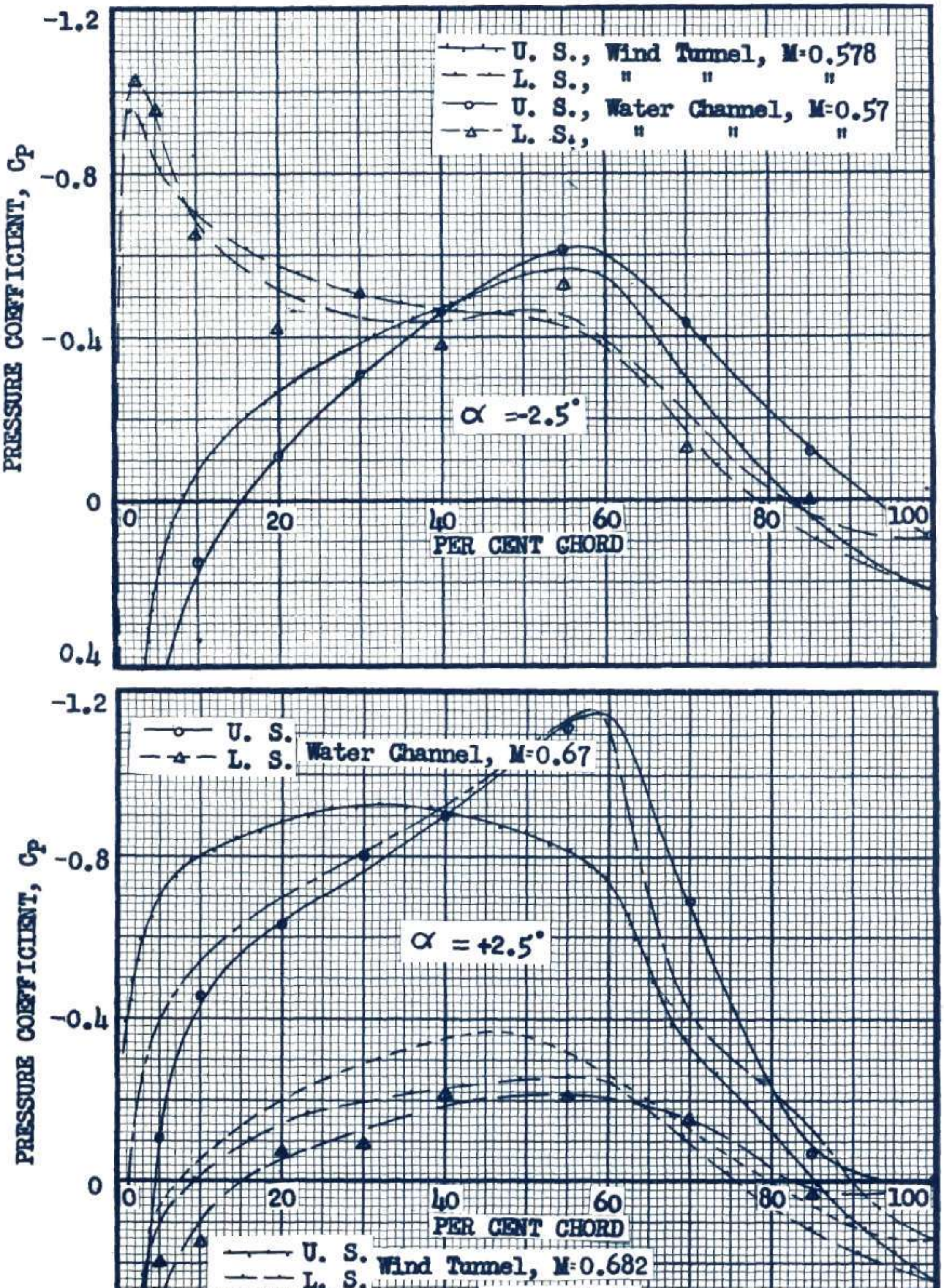



Fig. 13 COMPARISON OF WIND TUNNEL AND WATER CHANNEL PRESSURE DISTRIBUTIONS, GOLDSTEIN 1442/1547

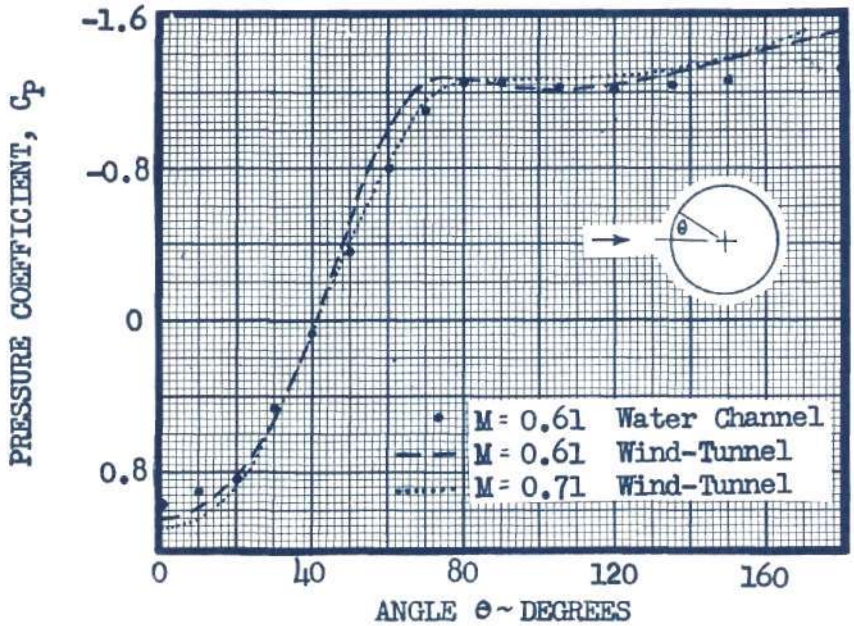



Fig. 14 PRESSURE DISTRIBUTION OF A CYLINDER (REF. 12)

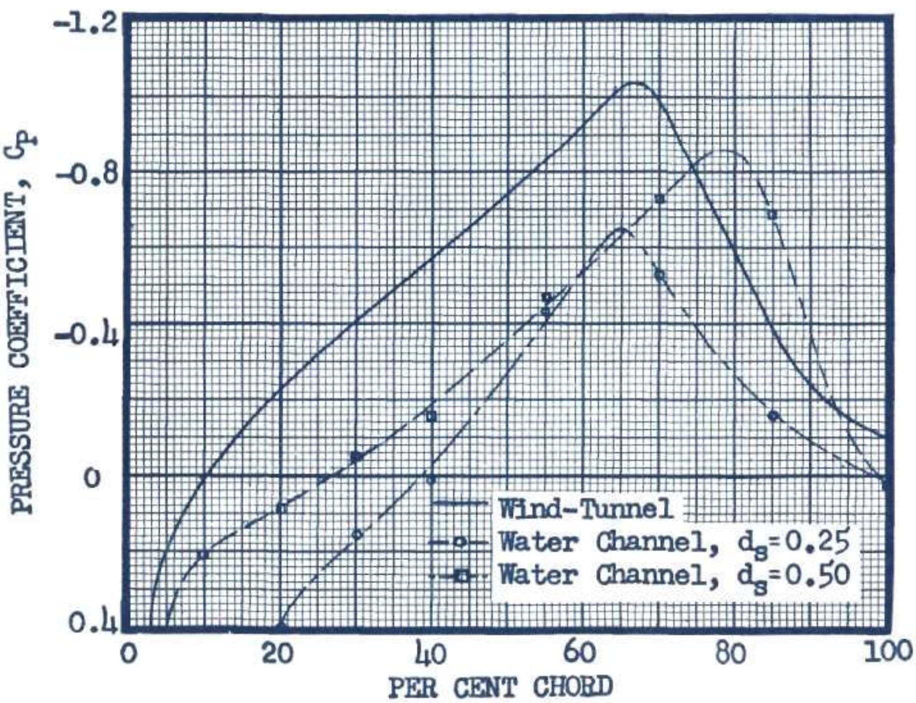



Fig. 15 EFFECT OF FREE STREAM DEPTH, MACH NUMBER = 0.85
GOLDSTEIN 1442/1547

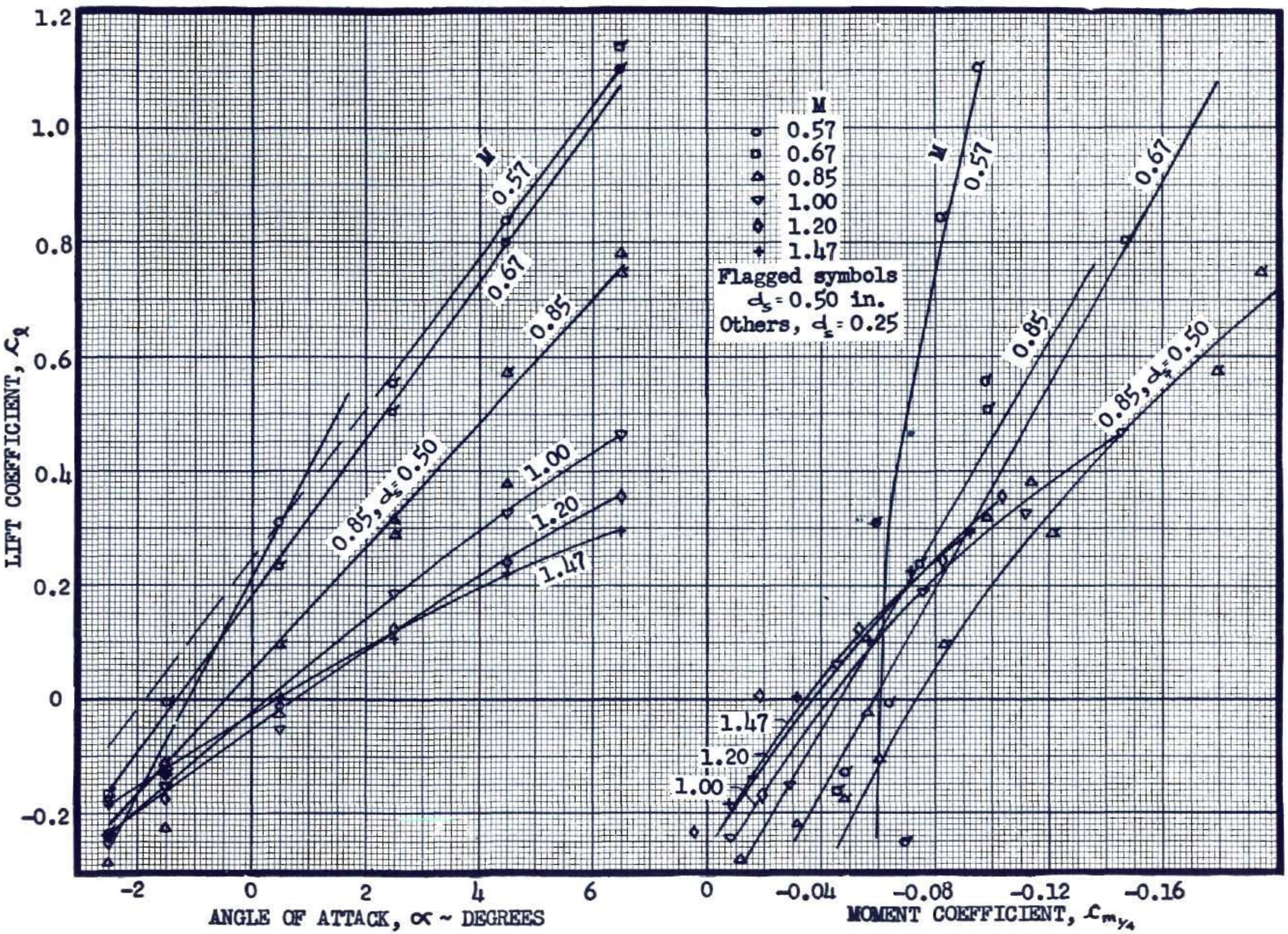



Fig. 16 LIFT AND MOMENT COEFFICIENTS, GOLDSTEIN 1442/1547

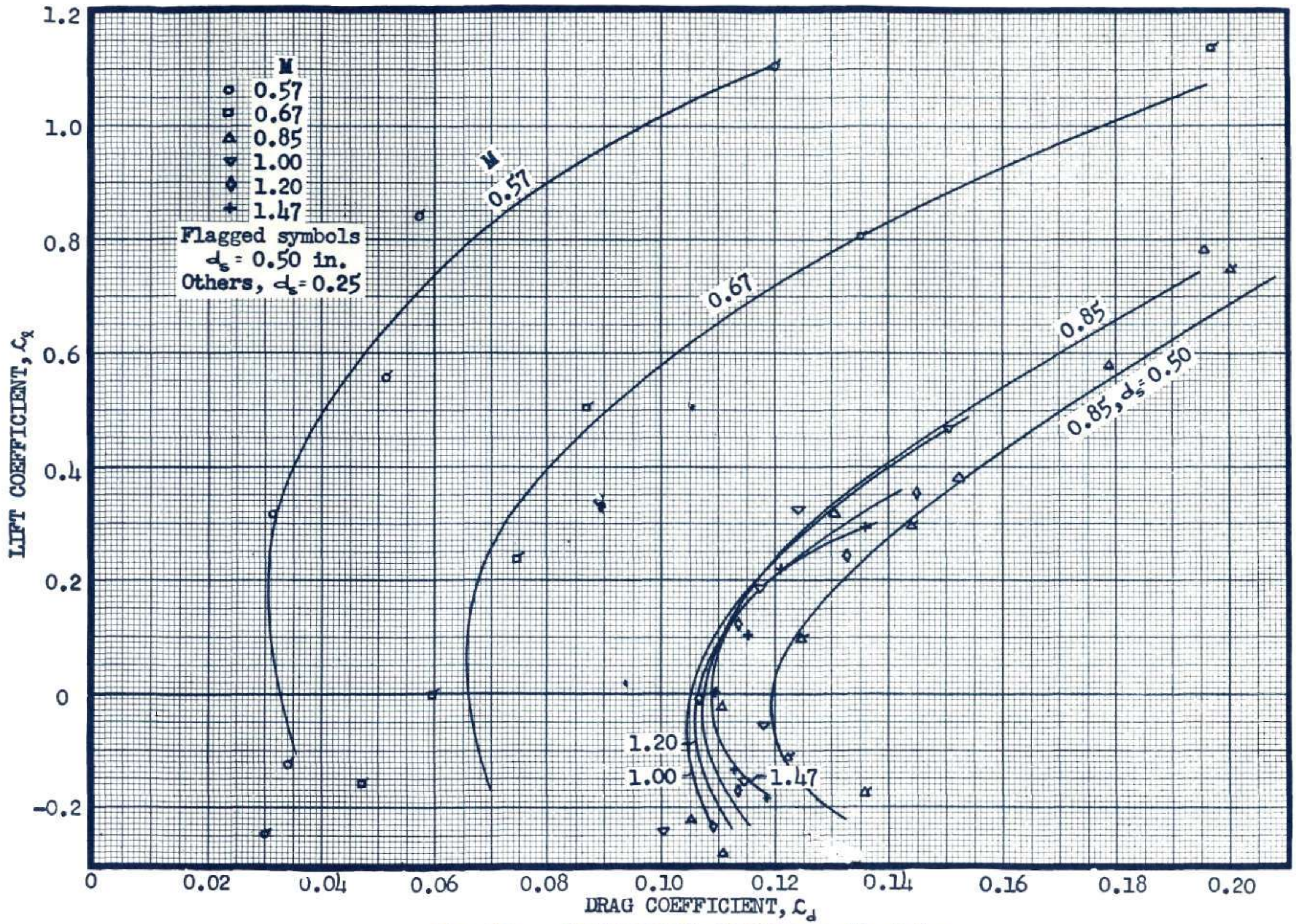



Fig. 17 DRAG POLARS, GOLDSTEIN 1442/1547

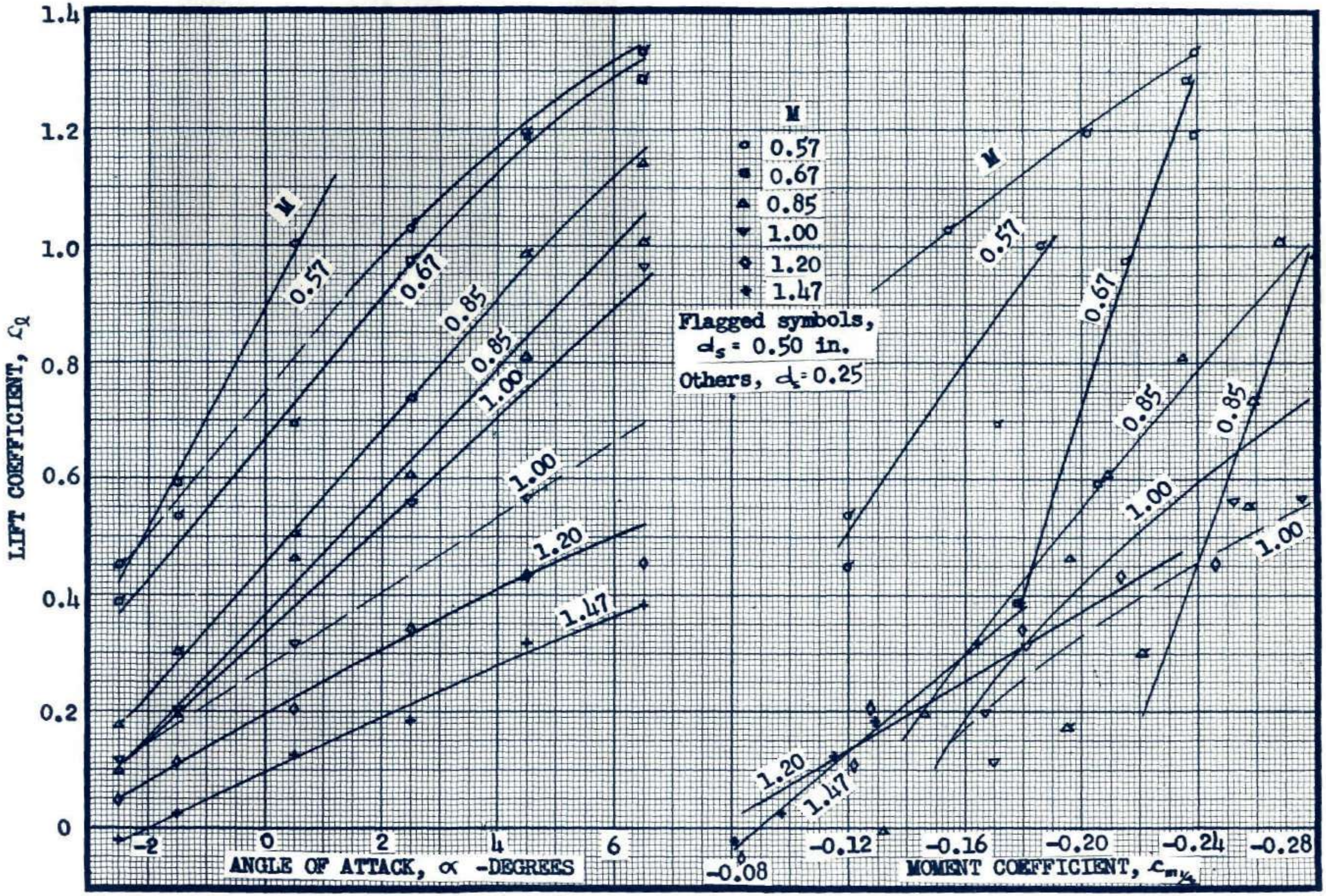



Fig. 18 LIFT AND MOMENT COEFFICIENTS, GÖTTINGEN 624

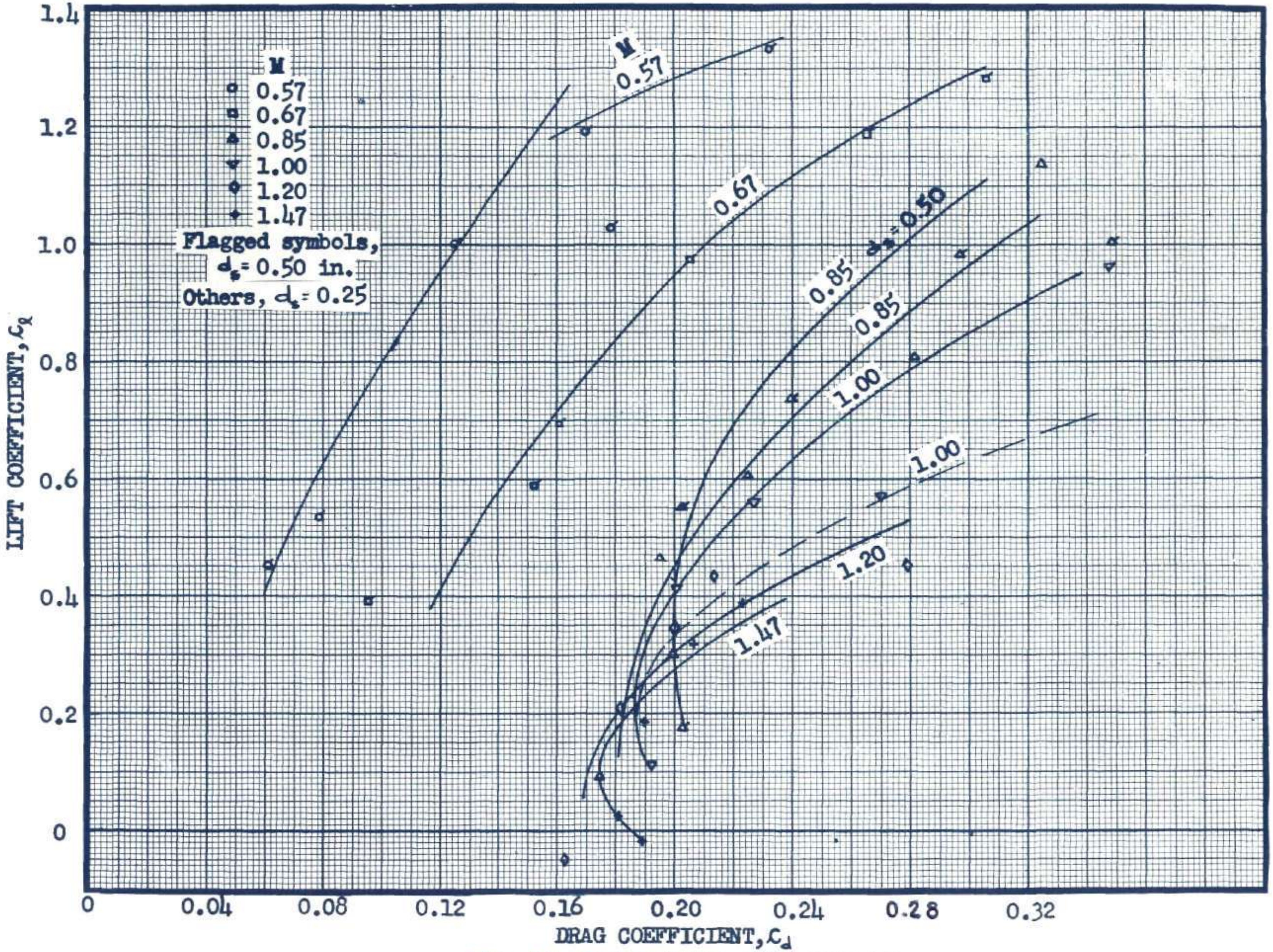
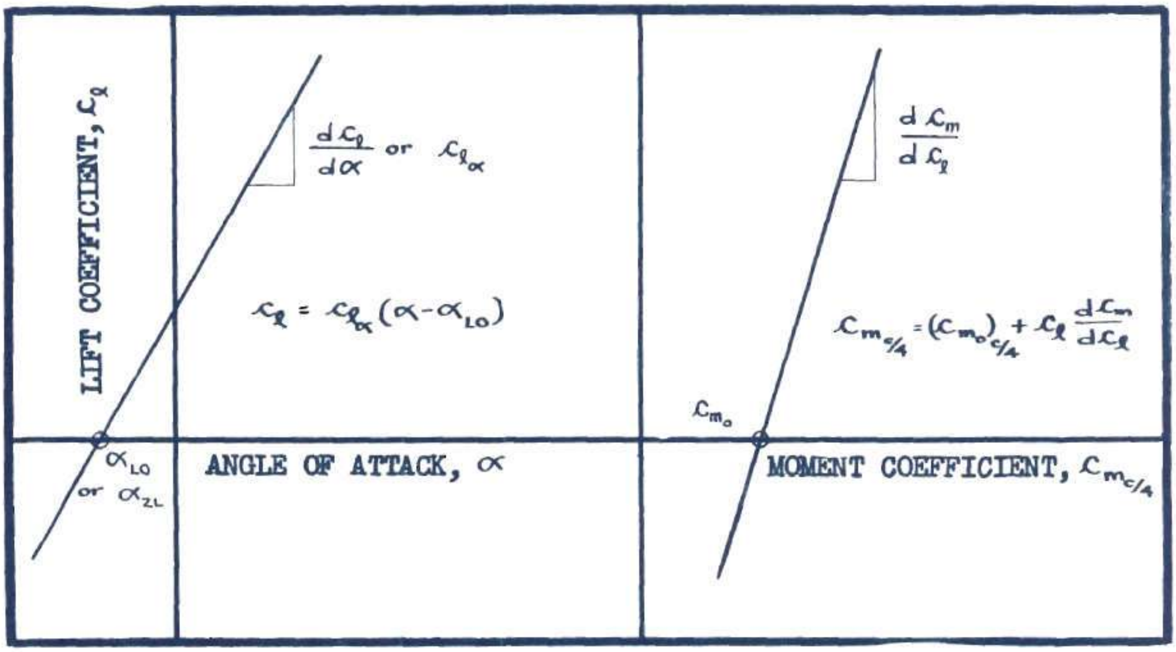
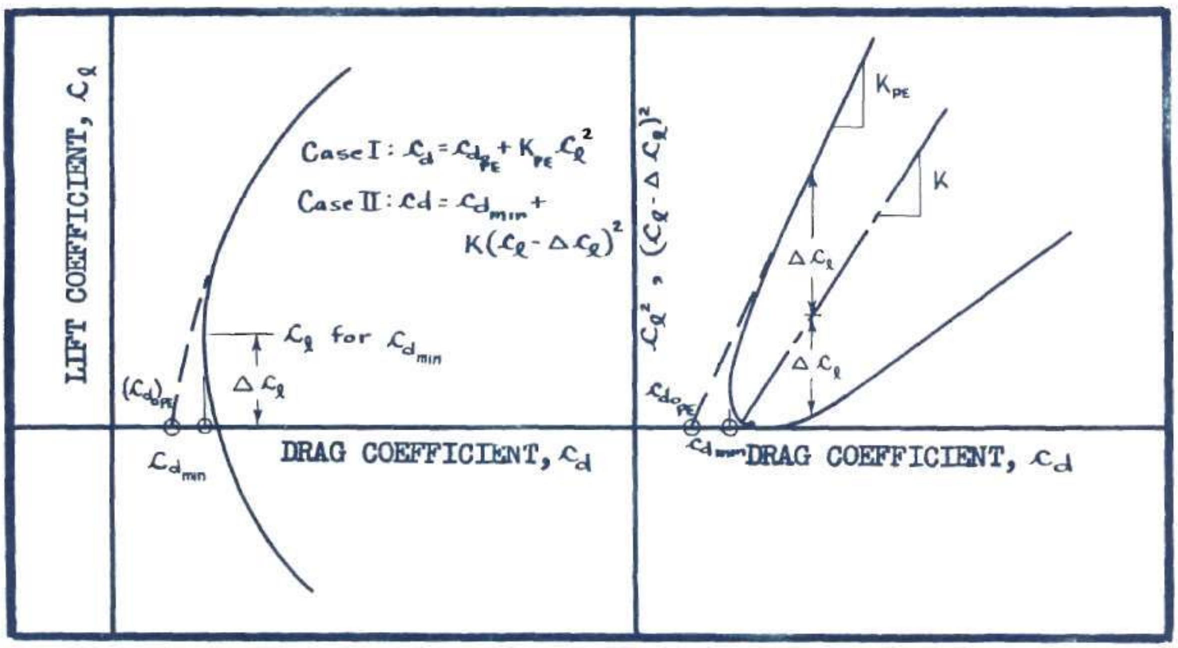



Fig. 19 DRAG POLARS, GÖTTINGEN 624




LIFT AND MOMENT PARAMETERS




DRAG POLAR PARAMETERS

Fig. 20 IDEALIZED AIRFOIL DATA PARAMETERS

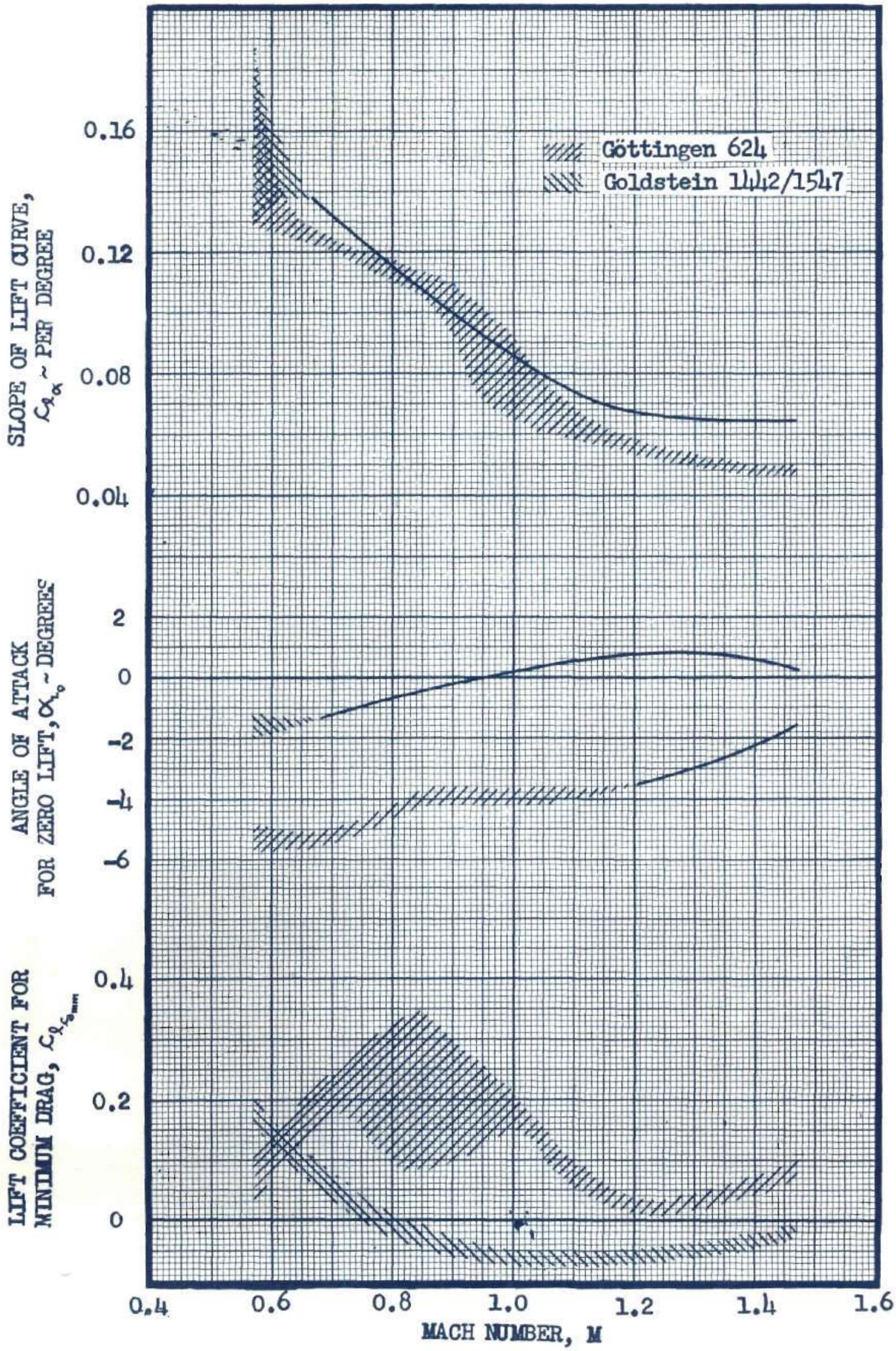



Fig. 21 COMPARISON OF LIFT PARAMETERS

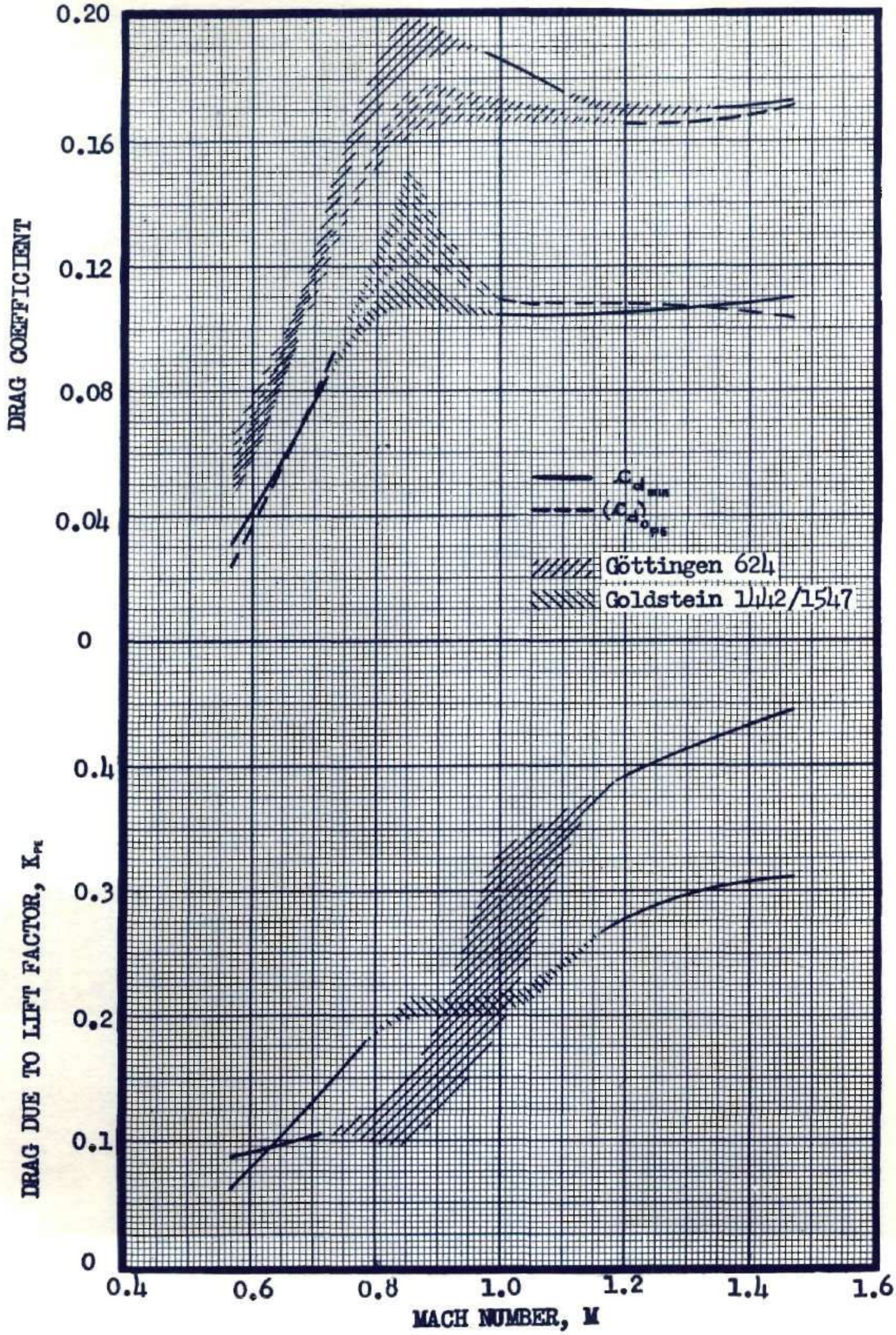



Fig. 22 COMPARISON OF DRAG PARAMETERS

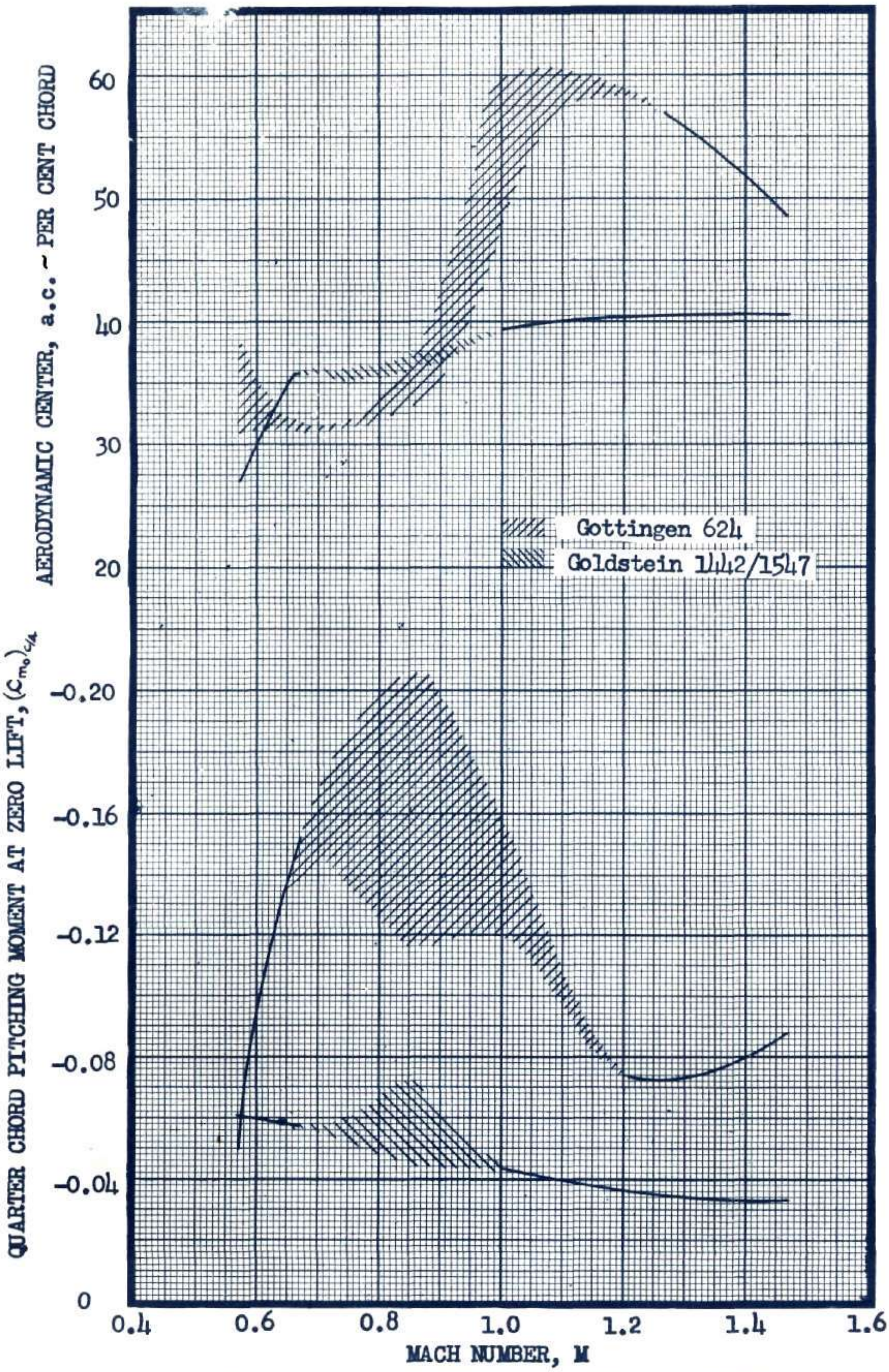
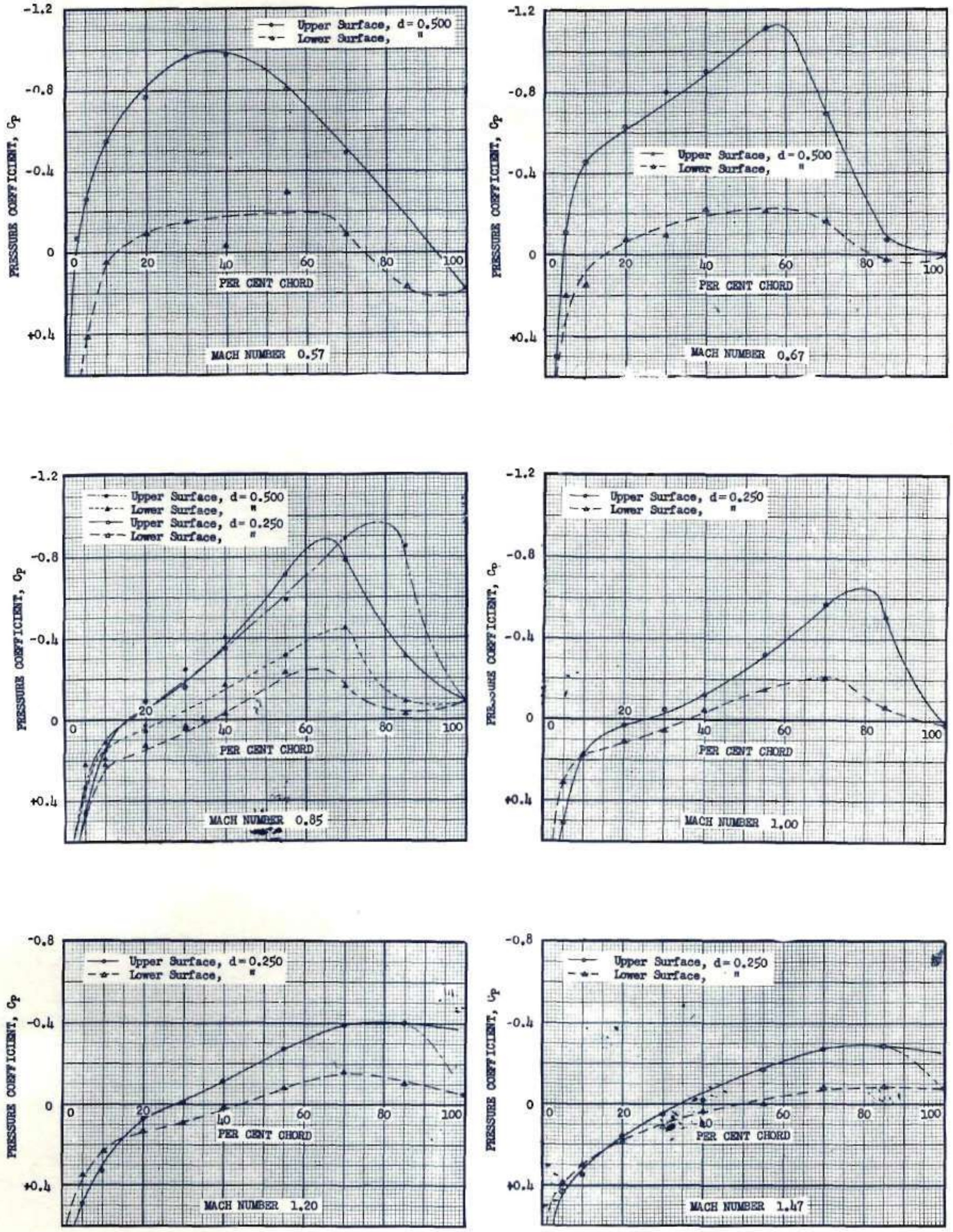
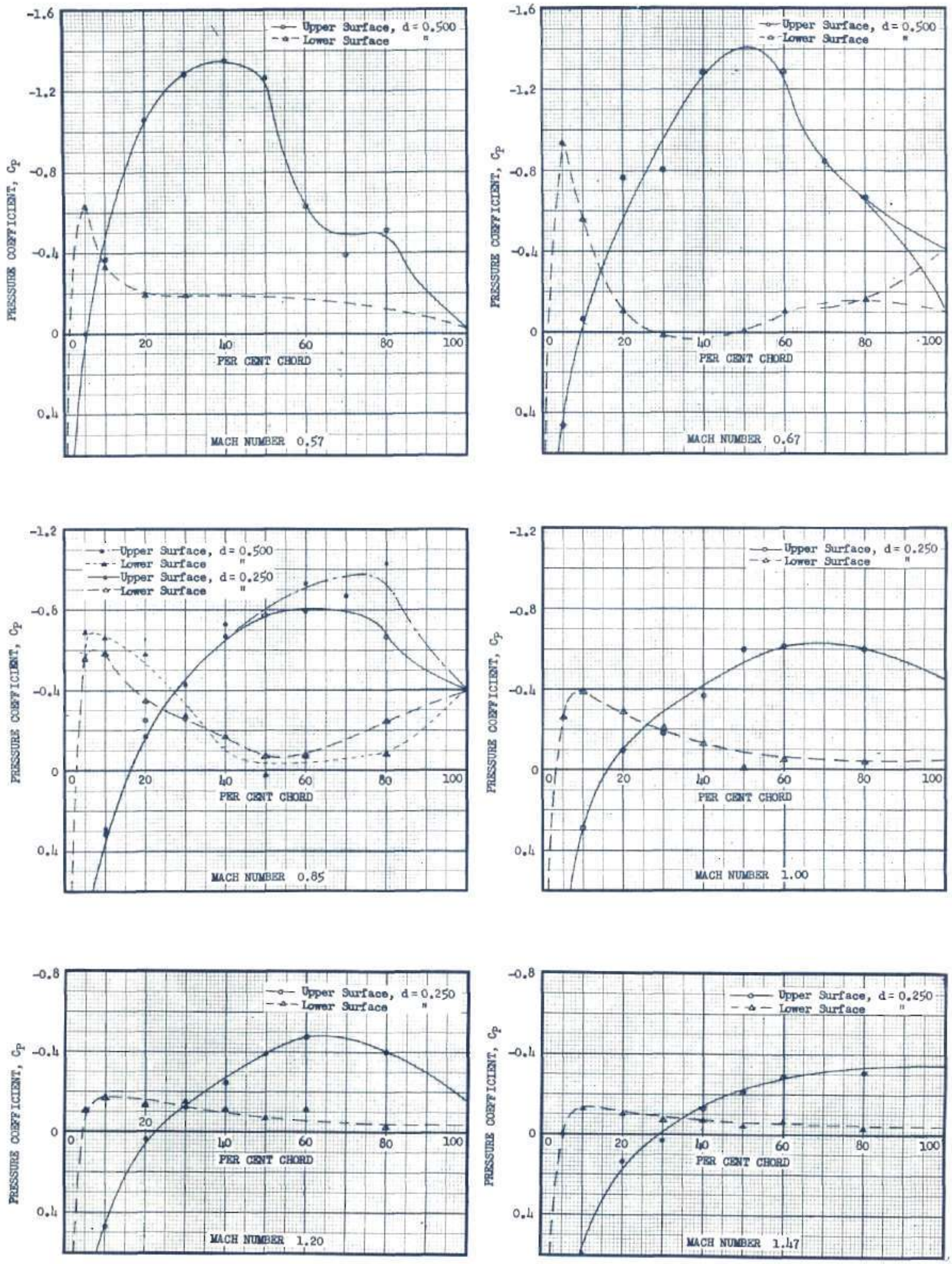



Fig. 23 COMPARISON OF MOMENT PARAMETERS




GOLDSTEIN 1442/1547, $\alpha$=2.5°

Fig.24 VARIATION OF PRESSURE DISTRIBUTION WITH MACH NUMBER




GÖTTINGEN 624, $\alpha = -1.5°$

Fig. 25 VARIATION OF PRESSURE DISTRIBUTION WITH MACH NUMBER

# BIBLIOGRAPHY

## LITERATURE CITED

1. von Karman, T., "Compressibility Effects in Aerodynamics," Journal of the Aeronautical Sciences, Vol. 8, No. 9, July 1941, pp 337-356.

2. Kaplan, C., On Similarity Rules for Transonic Flows, NACA TR 894, 1948.

3. Busemann, A., Application of Transonic Similarity, NACA TN 2687, April 1952.

4. Liepmann, H. W., "The Interaction Between Boundary Layer and Shock Waves in Transonic Flow," Journal of the Aeronautical Sciences, Vol. 13, No. 12, December 1946.

5. Van Dyke, M. D., High Speed Subsonic Characteristics of 16 NACA 6 Series Airfoil Sections, NACA TN 2670, March 1951.

6. Jouquet, E., "Quelques problems d'hydrodynamique g'ene'ral," Journal des Mathematiques Peres et Applique es, (8), T.111, 1920.

7. Riabouchinsky, D., "Mecanique des fluides," Comptes Rendus, t.195, No. 22, Nov. 28, 1932, pp 998-999.

8. Riabouchinsky, D., "Mecanique des fluides," Comptes Rendus, t.199, No. 14, Oct. 1, 1934, pp 632-634.

9. Binnie, A. M., and S. G. Hooker, "The Flow under Gravity of an Incompressible and Inviscid Fluid through a Constriction in a Horizontal Channel," Proceedings of the Royal Society (London, Ser. A, Vol. 159, No. 899, April 1937, pp 592-608.

10. Preiswerk, E., Application of the Methods of Gas Dynamics to Water Flows with Free Surface

    Part I Flows with No Energy Dissipation, NACA TM 934, March 1940.

    Part II Flows with Momentum Discontinuities (Hydraulic Jumps) NACA TM 935, March 1940.

11. Orlin, W. J., J. J. Lindner and J. G. Bitterly, Application of the Analogy Between Water Flow with a Free Surface and Two-Dimensional Compressible Gas Flow, NACA TR 875, 1947.

12. Laitone, E. V., A Study of Transonic Gas Dynamics by the Hydraulic Analogy, Office of Naval Research TR HE 116-323, Series 3, Issue 3, June 1951.

13. Harleman, D.R.F., Studies of the Validity of the Hydraulic Analogy to Supersonic Flow, Parts I and II USAF TR 5985, May 1950.

14. Hatch, J. E. Jr., Application of the Hydraulic Analogy to Problems of Two-Dimensional Compressible Gas Flow, Unpublished M. S. Thesis, Georgia Institute of Technology, 1949.

15. Catchpole, E. J., Application of the Hydraulic Analogy to Study the Performance of Two Airfoils in Compressible Flow, Unpublished M. S. Thesis, Georgia Institute of Technology, 1949.

16. Pearch, H. H. and J. A. Beavan, Force and Pressure Coefficients up to Mach Number 0.87 on the Goldstein Roof-Top Section 1442/1547. British Arc, Rand M 2346, 1950.

17. Busemann, A. and O. Walchner, "Profilergenschaften bei Ubershall-geschurdegkeit," Forshung auf dem Gebeite des Ingenieurwessens, March/April 1933, Berlin.

18. Milne-Thompson, L. M., Theoretical Hydrodynamics, London: Macmillan and Co., Limited, pp 355-356.

19. Allen, H. J. and W. G. Vincenti, Wall Interference in a Two-Dimensional Flow Wind-Tunnel with Consideration of the Effect of Compressibility, NACA TR 782, 1944.

20. Ryle, D. M., Jr., Application of the Hydraulic Analogy to Study the Performance of Three Airfoils at Subsonic and Supersonic Speeds, Unpublished M. S. Thesis, Georgia Institute of Technology, 1950.

21. Ferri, A., Method for Evaluating from Shadow or Schlieren Photographs the Pressure Drag in Two-Dimensional or Axially Symmetrical Flow Phenomena with Detached Shock, NACA TN 1808.

# OTHER REFERENCES


## Hydraulic Analogy

Crossley, H. E., Jr., The Analogy between Surface Shock Waves in a Liquid and Shocks in Compressible Gases, California Institute of Technology Report No. N-54.1, August 1949.

Gilware and Plesset, The Analogy between the Flow of a Liquid with a Free Surface and the Two-Dimensional Flow of a Gas, California Institute of Technology Memo Report No. M-54.1, March 1949.

Ippen, A. T. and R. T. Knopp, "Curvilinear Flow of Liquids with Free Surface at Velocities above That of Wave Propogation," Proceedings of the 5th International Congress of Applied Mechanics, 1938, pp 531-536.

Johnson and Witbeck, Water Analogy to Two-Dimensional Airflow, General Electric, Theoretical Transonic and Supersonic Flow, Report No. 55218, 1941.

Thomas, G. A., Application of the Hydraulic Analogy to Study the Performance of Two Airfoils in Compressible Flow, Unpublished M. S. Thesis, Georgia Institute of Technology, 1949.

## Theoretical Transonic and Supersonic Flow

Ackeret, J., F. Feldman and N. Roth, Investigations of Compression Shocks and Boundary Layer in Gases Moving at High Speeds, NACA TM 1113, January 1947.

Busemann, A., A Review of Analytical Methods for the Treatment of Flows with Detached Shocks, NACA TN 1858, April 1949.

Ferri, A., Application to Aeronautics of Ackerets Theory of Aerofoils Moving at Speeds Greater than That of Sound, British ARC R & M 1467, 1932.

Gothert, B., and K. H. Kawalki, The Compressible Flow Past Various Plane Profiles near Sonic Velocity, NACA TR 1203.

Guderley, Gottfried, "New Aspects of Transonic Flow Theory," Technical Data Digest, Vol. 12, No. 9, 1 November 1947, pp 105-129.

Liepmann, H. W., H. Askenas and J. D. Cole, Experiments in Transonic Flow, USAF Technical Report No. 5667, 19 February 1948.

Lin, C. C. and S. T. Rubinov, "On the Flow Behind Curved Shocks," Journal of Mathematics and Physics, Vol. XXVII, No. 2, July 1948, pp 105-129.

Nitzberg, G. E. and S. Crandall, Some Fundamental Similarities between Boundary-Layer Flow at Transonic and Low Speeds, NACA TN 1623 June 1948.

Perl, W., Calculation of Transonic Flows Past Thin Airfoils by an Integral Method, NACA TN 2130, July 1950.

Shu, S. S., On Two-Dimensional Flow After a Curved Stationary Shock (with special reference to the problem of detached shock waves) NACA TN 2364, May 1951.

Tsien, H-S, The "Limiting Line" in Mixed Subsonic and Supersonic Flow of Compressible Fluids, NACA TN 961, November 1944.

Tsien, H-S, and A. Fejer, A Method for Predicting the Transonic Flow over Airfoils and Similar Bodies from Data Obtained at Small Mach Numbers, CADO ATI No. 16128, 31 December 1944.

Tsien, H-S, and Y-H Kuo, Two Dimensional Irrotational Mixed Subsonic and Supersonic Flow of a Compressible Fluid and the Upper Critical Mach Number, NACA TN 995, May 1946.

Vincenti, W. G., and C. B. Wagoner, Transonic Flow Past a Wedge Profile with Detached Bow Wave - General Analytical Method and Final Calculated Results, NACA TN 2339, April 1951.

von Karman, T., "Supersonic Aerodynamics - Principles and Applications," Journal of the Aeronautical Sciences, Vol. 14, No. 7, July 1947.

Kuo, Y-H, Two-Dimensional Irrotational Transonic Flows of a Compressible Fluid, NACA TN 1445, June 1948.

Wind-Tunnel Testing

Daley, B. N. and M. D. Humphreys, Effects of Compressibility on the Flow Past Thick Airfoil Sections, NACA TN 1657, July 1948.

Emmons, H. W., Flow of a Compressible Fluid Past a Symmetrical Airfoil in a Wind-Tunnel and in Free Air, NACA TN 1746, November 1946.

Lindsey, W. F., Effect of Compressibility on the Pressures and Forces Acting on a Modified NACA 65,3-019 Airfoil Having a 0.20 - Chord Flap, NACA ACR L5G31a (Declassified) January 1946.

Lock, C. N. H. and J. A. Beaven, Tunnel Interference at Compressibility Speeds Using the Flexible Walls of the Rectangular High Speed Tunnel, British ARC R & M 2005, 27 September 1944.

Weaver, J. H., "A Method of Wind-Tunnel Testing through the Transonic Range," Journal of Aeronautical Sciences, January 1948.

General

Ferri, A., Elements of Aerodynamics of Supersonic Flows, New York: The Macmillan Co., 1948.

Handbook of Supersonic Aerodynamics, NAVORD Report 1488 (Vol. 1) Washington U. S. Government Printing Office, 1 April 1950.

Kuethe, A. M. and J. D. Schetzer, Foundations of Aerodynamics, New York John Wiley and Sons, Inc., 1950.

Liepmann, H. W. and A. E. Puckett, Introduction to Aerodynamics of a Compressible Fluid, New York, John Wiley and Sons, 1947.

Sibert, H. W., High-Speed Aerodynamics, New York: Prentice-Hall, Inc., 1948.